THEORY AND SIMULATION OF FIELD EMISSION FROM MICROSTRUCTURES

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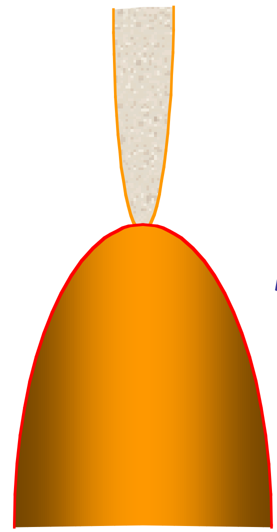
DISCLAIMER

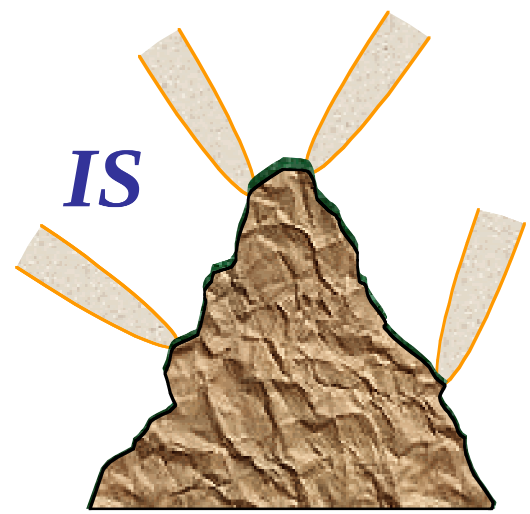
The Theoretical Modeling of Field Emission Structures Herein Presuppose Uniform, Smooth Structures Amenable to Analytical Analysis and Numerical Simulation.



SHOULD

Nature Is More Complicated Than That.

IS



Warning: Use of These Theories May Induce Drowsiness, Dizziness, and Blurry Vision

OUTLINE

Introduction:
- What Are Field Emitters?
- What Makes Them Attractive?
- Where Can They Be Used?
- What Are the Demands On Them?

Theory of Field Emission (1-D)
- The Surface Barrier
- Transmission of Electrons
- Supply of Electrons
- The 1-D Fowler Nordheim Eq.
- Metal vs. Semiconductor

The Statistical Hyperbolic Model
- 3-D Emitter Structures
- Area Factors / Field Enhancement
- Statistics
- Using the Model

Relation of Model to Experiment
- I-V Measurements and Modeling
- Emission Distribution Experiment
- Space Charge

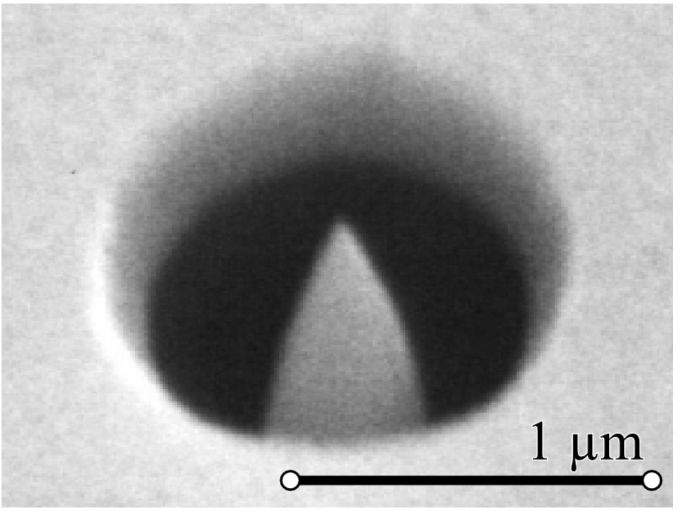
Interpretation of Experimental Data
- Different Geometries
- Different Test Stations
- Conditioning
- Effects of ZrC Coating

Concluding Remarks

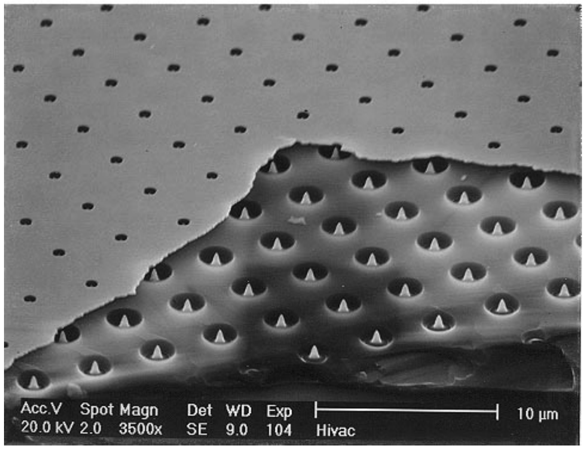
WHAT ARE FIELD EMITTER ARRAYS?

FEA's:

- Are Microfabricated Field Emission Structures Developed Using Thin Film Technology and Processing Techniques
- Have Set Record Current Density Levels for Any Cathode (>2000 A/cm^2)
- Utilize Intense Electric Fields Generated From Micron-scale, Conical, Gated Structures
- Are Instant ON/OFF Cold Cathodes

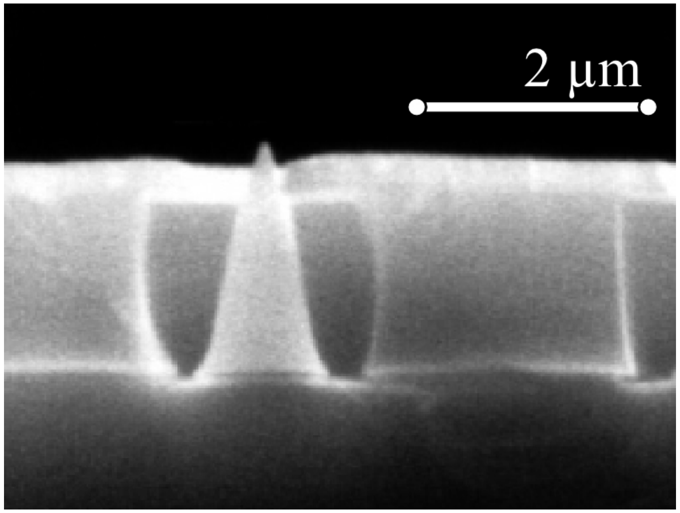


P{hotographs courtesy of C. Spindt (SRI)



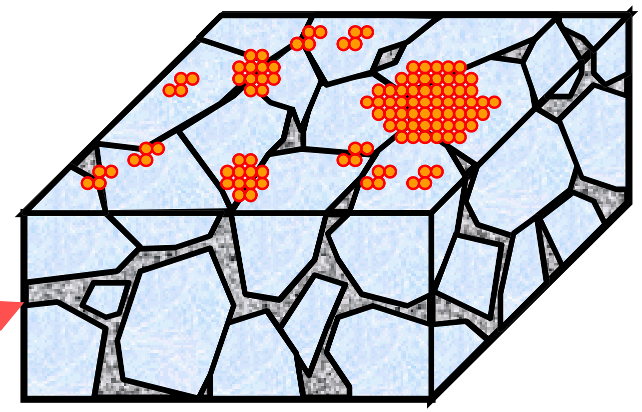
Shown to Left:
Typical layout and dimensions of a field emitter array

Shown to Right:
***Comparable FEA Geometry & Size to Single Tips Operated at** *100 µA**

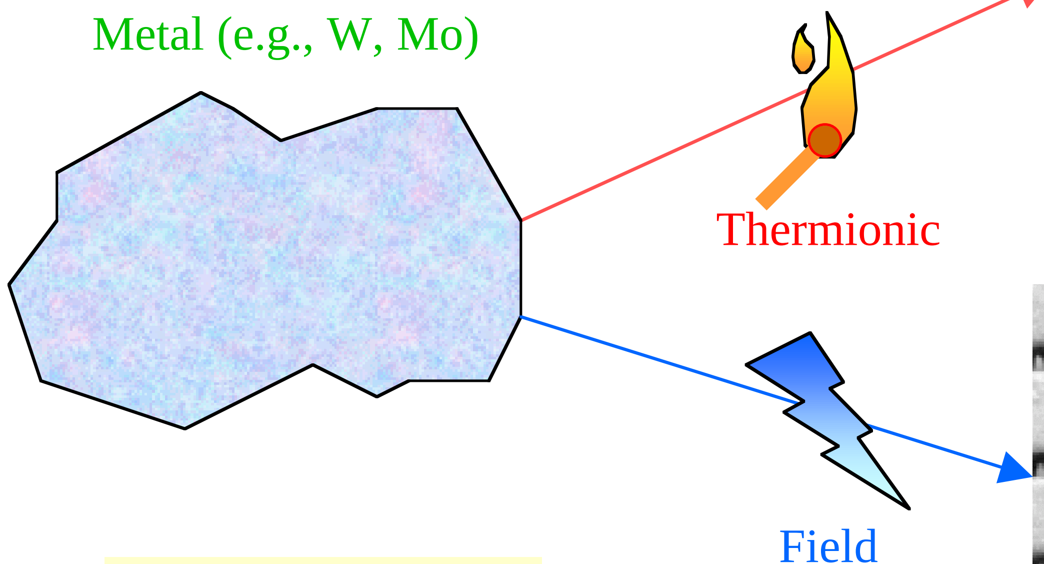


THERMIONIC AND FIELD EMISSION

The Manner in Which Electrons Are Extracted Dictates the Technological Gambits Invoked



Low Work Function Islands (e.g., Ba)



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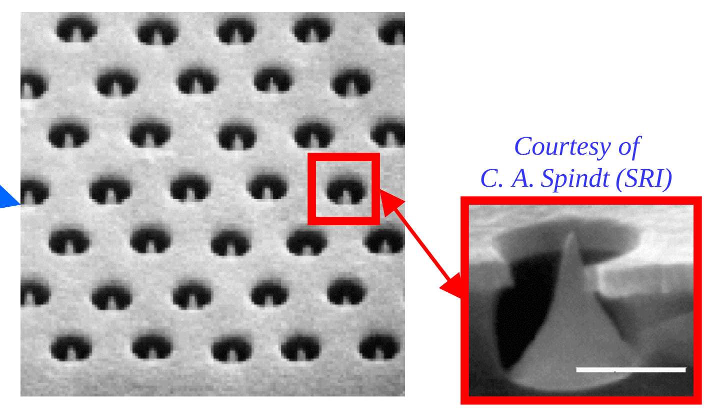
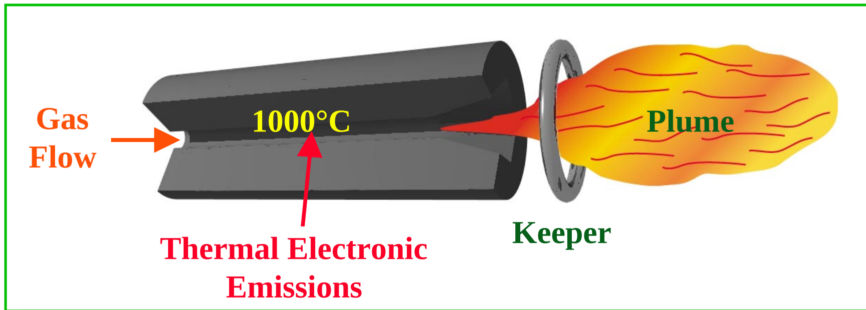


Geometric Field Enhancement

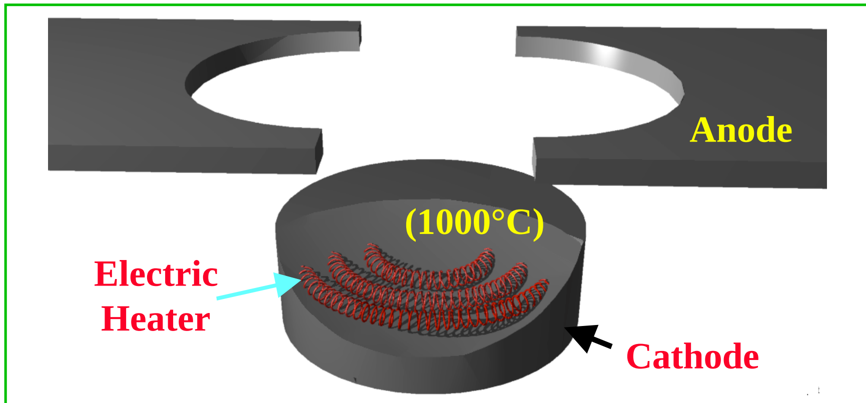
Other Emission Methods
- **Secondary**
- **Piezoelectric**
- **Photoemission**

CATHODE COMPARISON



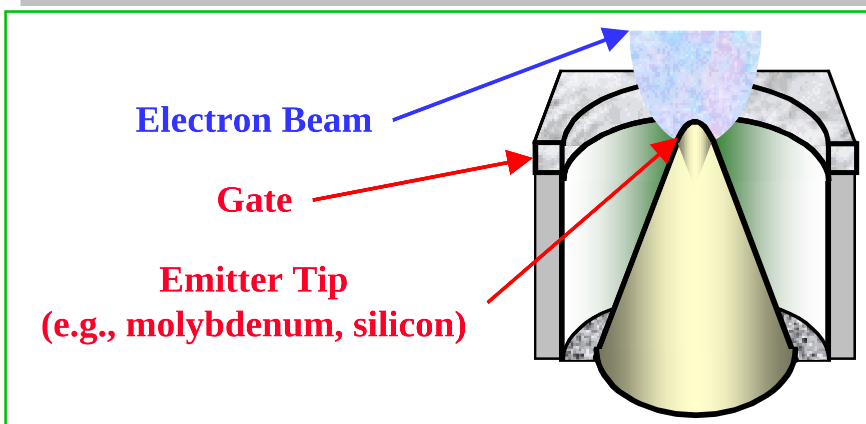
HOLLOW CATHODE

- Gas Flows Through Heated Tub; Electrons Thermally Emitted, Accelerated By Keeper. Gas Nearly Fully Ionized By Electrons. Expanding Plasma Provides High Conductivity Connection w/ Space Plasma
- Reliability Demonstrated on Multiple Space Flights;
- **Requires High Power and Weight**



THERMIONIC EMITTER

- Electrons Ejected Thermionically From Heated Cathode. Electrons Accelerated Away by Anode
- Mature Technology That Has Been Used for Decades in Vacuum Tubes
- **High Current Density at Expense of Lifetime; Space Applications at 200 km Atmospheric Pressure is Near Operational Limit**



FIELD EMITTER ARRAY

- Application of Voltage to Microfabricated Micron-scale Conical Emitter Creates Electric Fields ≥ kV/µm; Enables High Current Densities Due to Tunneling.
- Cold Emission, No Heating Required, Instant On-off Performance, Complete Suppression of Emission Current With Small Change in Gate Voltage
- **Further Application Specific Development Required**

HEIRARCY OF APPLICATIONS

>10 A/cm^2

High Modulation

Emission Gated Amplifiers
Radar
Communications
Electronic Warfare

Space-based Applications
(Thrusters, Tethers, Discharge, etc.)

Electron Sources
X-ray Sources
Electron Beam Lithograpy
Flat Panel Displays

Mass Spectrometers
Scanning Electron Microscopes (SEMs)
Chemical Analysis Spectrometers

< 10 mA/cm^2

Low or No Modulation

RF AMPLIFIER DEMANDS ON CATHODES

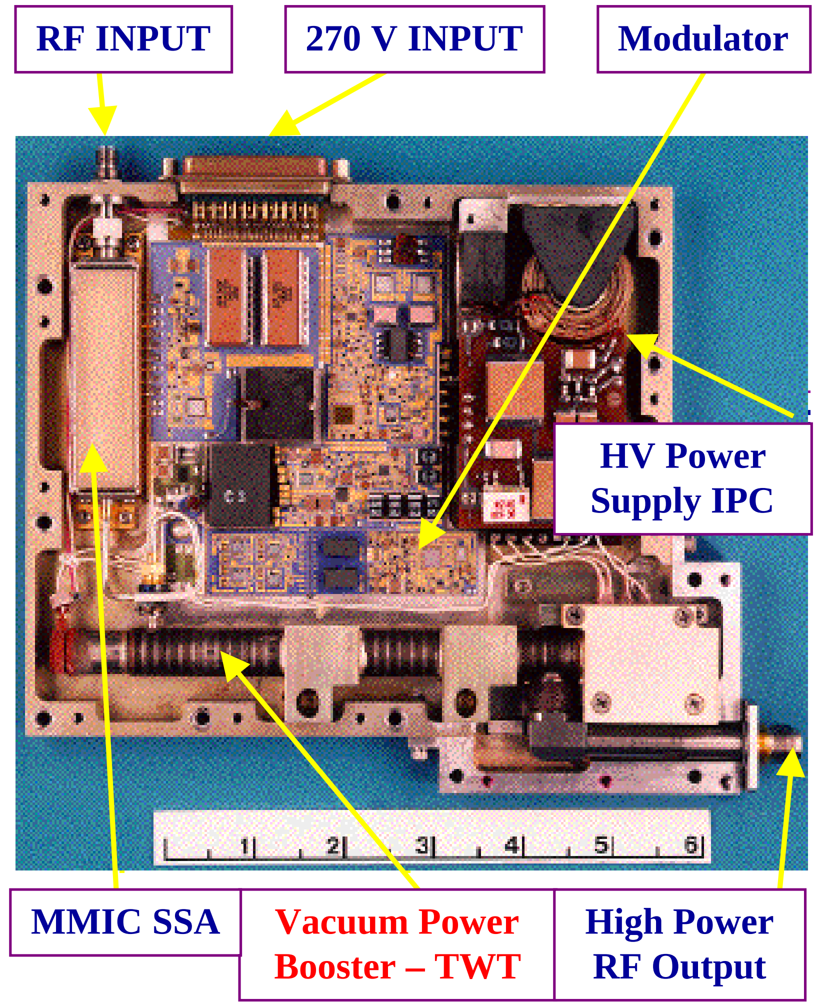
THERMIONIC EMITTERS:

- Increase T to Increase J, But Lifetime Decreases
- $J \leq 5$ A/cm^2 ; Beam Convergence of 30-50:1 Required; Exotic Devices Require >1000:1
- Large Beams & Sophisticated Gun Designs w/ Highly Convergent B Fields Required.
- Gridded Cathodes Transit Time Limit ≈ 2 GHz
- Velocity Modulation of Beam: Majority of Circuit Used for Bunching

FIELD EMITTER ARRAYS

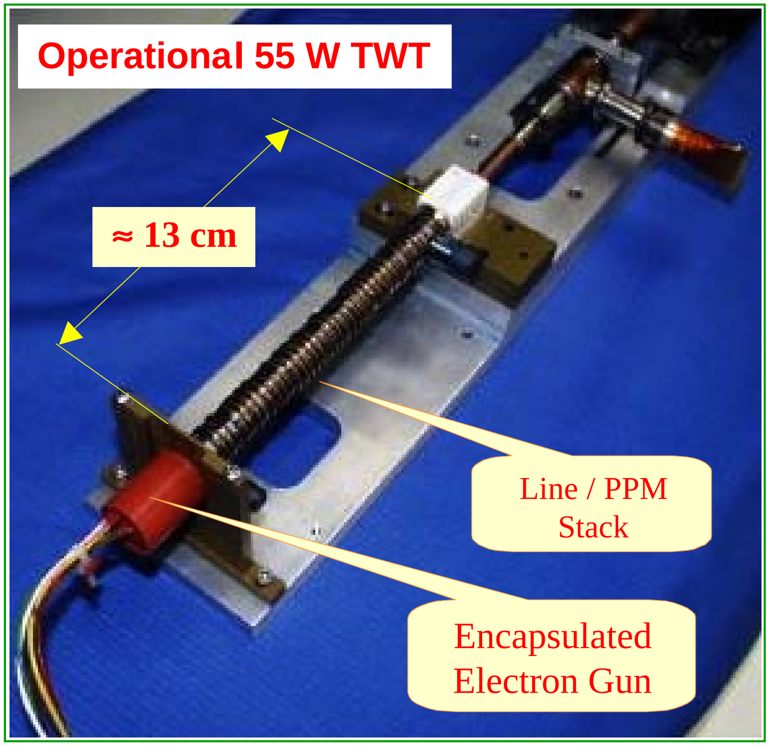
- High J (>500 A/cm^2) Relaxes Convergence Factor, Simplifies B Profile, Relaxes Machining Tolerances, Reduces Beam Scalloping and Beam Interception by Circuit (Helix)
- Decrease in Weight, Volume, Power Consumed
- Transit Time ≈ 1 THz: Emission-Gating

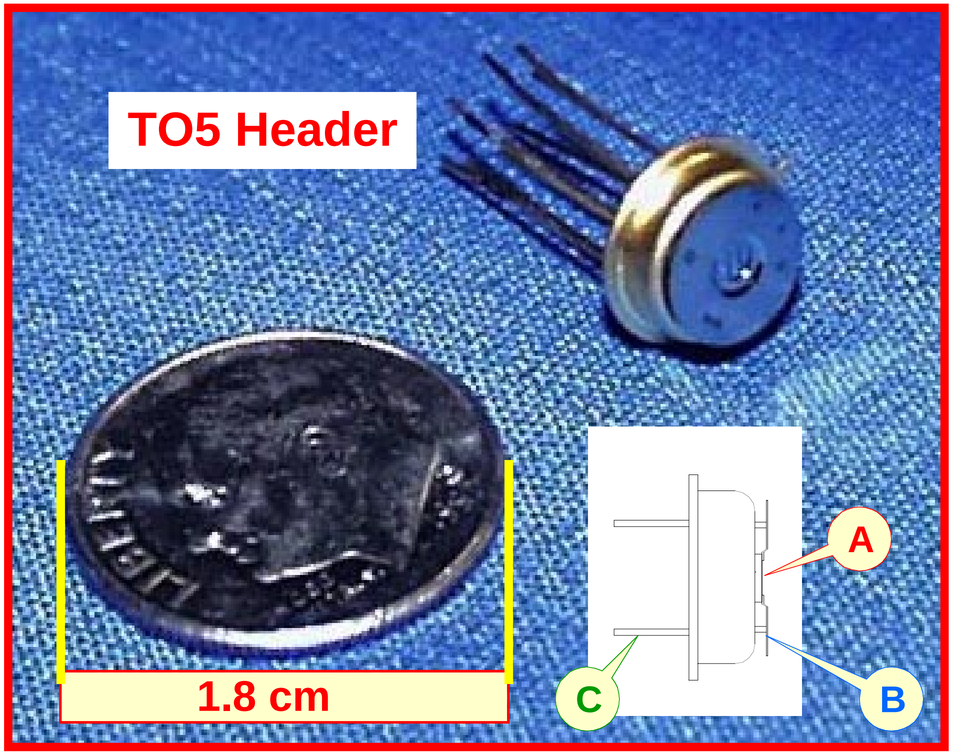
Microwave Power Module



FEA CATHODE FOR TWT

Photos courtesy of David Whaley (Northrop Grumman)





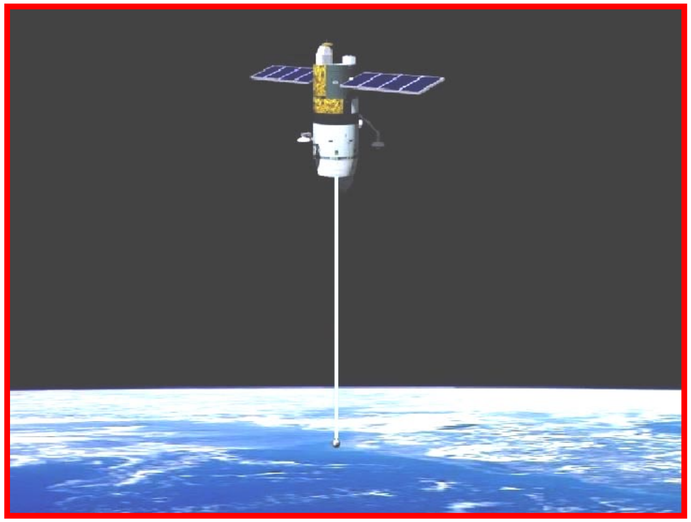
94.1 mA From Area of Diameter ≤ 1 mm With 50,000 Tips

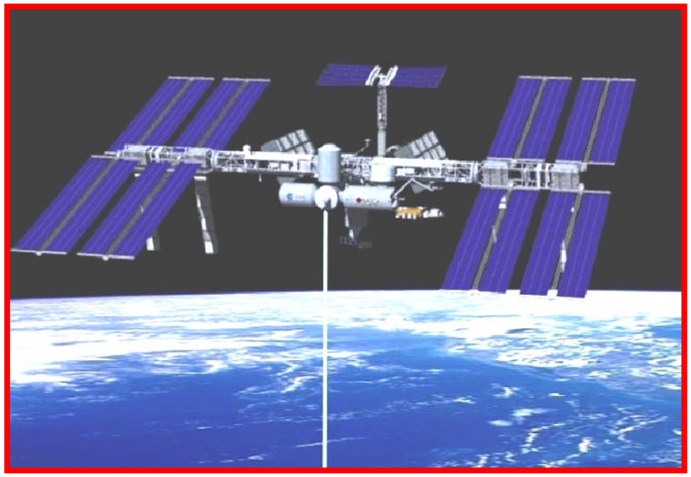
D. Whaley, *et al*., IEEE-TOPS28, 727 (2000)

A - Field Emitter Array
B - Gate Hold-Down Disk
C - Base/Gate Leads

ELECTRODYNAMIC (ED) TETHERS

- Provides Ability to Raise, Lower or Change Inclination of Spacecraft Orbit w/o Consumables
- Spacecraft Drag Make-up for Lifetime Extension
- Rapid and Frequent Maneuvers for Earth Observations Without Propellant and Limiting Spacecraft Lifetime
- Positioning of Distributed Spacecraft Constellations Without Propellant
- Spacecraft Tracking Avoidance by Regularly Changing Spacecraft Orbital Altitude/inclination
- Contamination-free Propulsion System
- Minimizes Cost by Allowing Launching Into Low Earth Orbit and Raising to a Higher Orbit Without Propellant





Courtesy Of Brian Gilchrist (U. Michigan)

ELECTRIC PROPULSION (EP)

FLEETS OF MICROSPACECRAFT

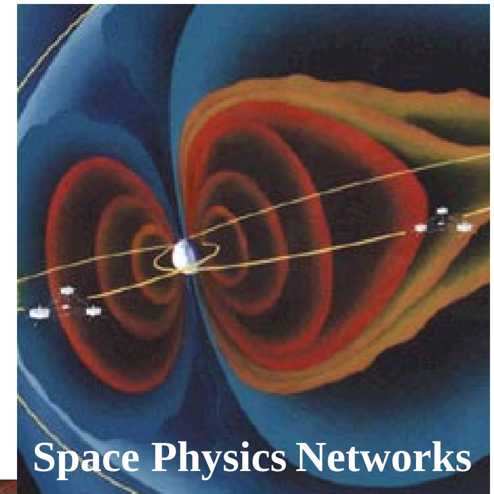
- Could Be Distributed by the Saturn Ring Observer With Micropropulsion Systems to Explore the Rings of Saturn.
- For the Space Physics Networks Measuring Fields and Particles.
- **For Distributed Inspector Spacecraft**

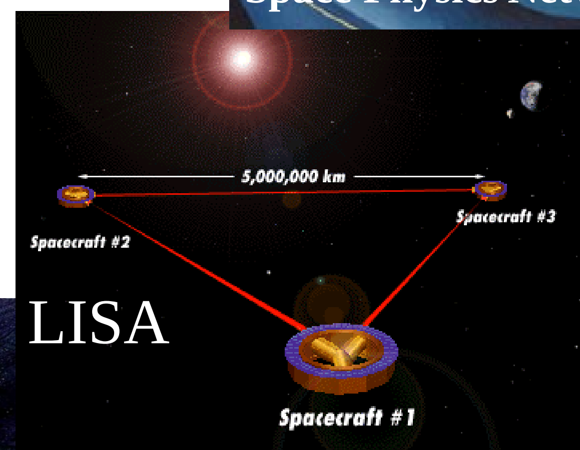
INFLATABLE SPACECRAFT

- Like ARISE for Detecting Black Holes Will Require Micropropulsion Systems to
- Compensate for Solar Disturbance Torques Induced by Its Large Inflatable Antenna.
- **For a Communication Relay and Signal Amplification Between Ground Stations and Microspacecraft**

CONSTELLATIONS OF SPACECRAFT

- For Formation Flying Interferometry Missions Like Laser Interferometer Space Antenna (LISA) and Terrestrial Planet Finder (TPF) Will Require Micropropulsion Systems for Relative Position Maintenance.
- **For 3-D Mapping of Surface Features Requiring High Precision Relative Spacecraft Position Maintenance**





Courtesy Of Colleen Marrese (JPL)

Electric Propulsion Systems Are Very Efficiently Employed for Low Thrust Maneuvers on Micro- to Large Inflatable Structures

FEAs IN FLAT PANEL DISPLAYS

Photo courtesy of Alec Talin (Motorola, Tempe AZ)

Photo taken at MRS Spring 2001 Symposium D

Motorola 15" Diag HV Field Emission Display
VGA (640x480) Res W/ 8 Bit/color
Emission Current of 2 μA/color Pixel
250 Tips / 1 Color Sub-pixel

Candescent's High Voltage
Field Emission Display DVD

Demonstrated by Chris Curtin,
Candescent Technologies, San Jose, CA

BEAM ON / OFF ISSUES

Beam Blanking (Turn e-Beam Off): $I_{min} \approx 0.1\%$ of I_{max}

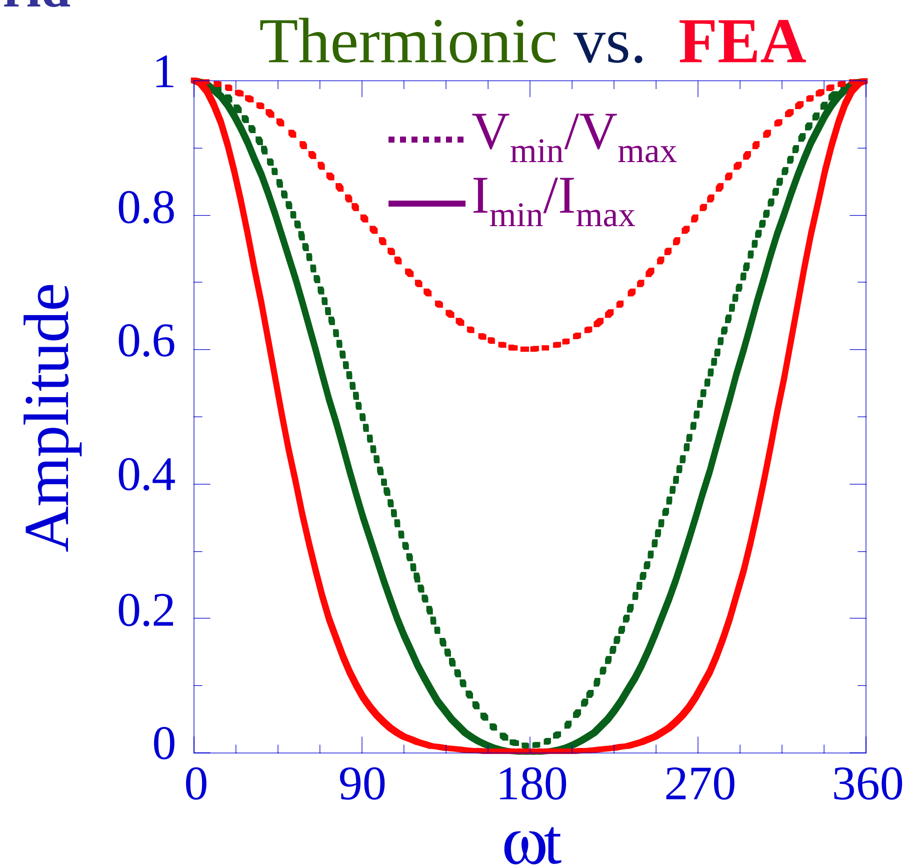
- **Reduction of kV-Voltage Swings Eases Demands on Solid State Power MOSFET Driver Used to Control Grid**

Thermionic Emitters:

- **Space Charge Limited Current:** $I(V) = P\,V_g^{3/2}$
- **Grid Voltage** $V_g \approx 1–10$ **kV**
- **Min Voltage ≈ 1% Max Voltage**

Field Emitter Arrays

- **Fowler Nordheim Current:** $I(V) = A\,V_g^2\,\mathrm{Exp}(-B/V_g)$
- **Grid Voltage** $V_g \approx 75$**V** ($B \approx 8\,V_g$)
- **Min Voltage ≈ 60% Max Voltage**



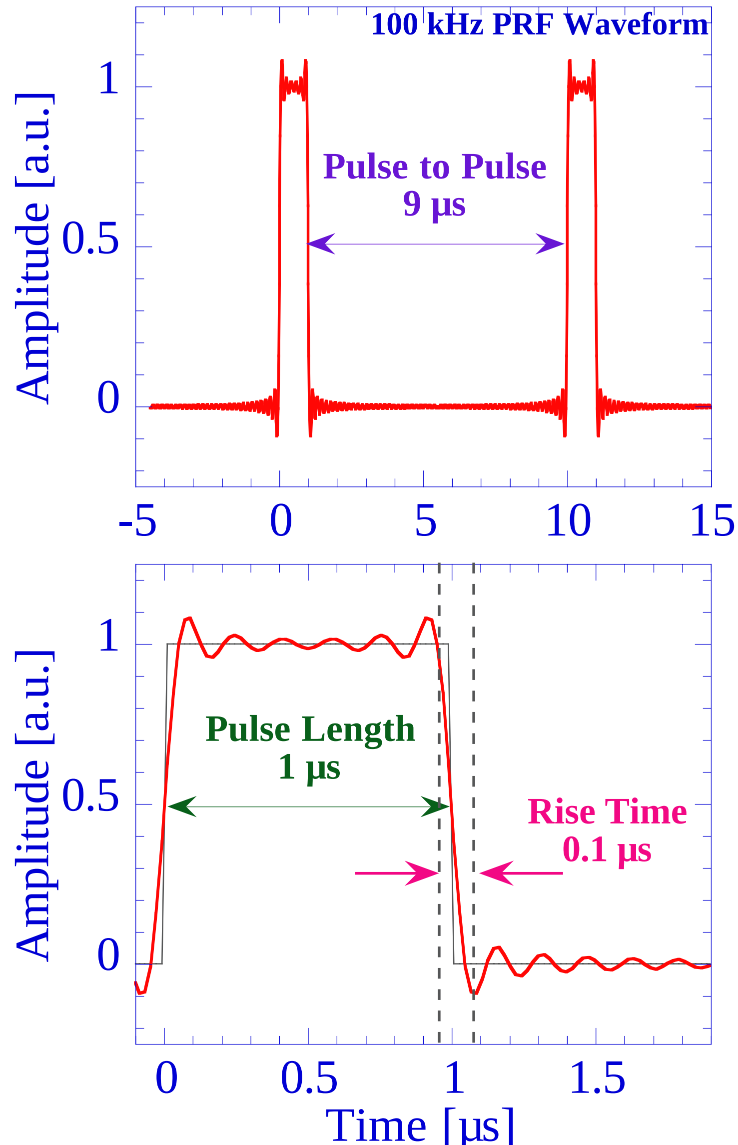
PULSE REPETITION FREQUENCY (PRF)

RADAR SYSTEMS UNDER DEVELOPMENT USING THERMIONIC EMITTERS:

- Required: PRFs of 100 kHz (100 ns rise time)
  Desired: PRFs of 1 MHz (10 ns rise time)
- Present Gridded Thermionic Sources:
  Pulse Rise Time Too Long: Larger Rise Times Shorten Pulse-to-Pulse Time, Decreases “Listen” Time Available for Return Signal (Pulse-to-Pulse Separation); Emission Noise Degrades Listening Window for Similar Reasons.

FIELD EMITTER ARRAYS:

- **10-ns Rise Time** ⇔ Modulation @ 0.05 GHz.
- In Klystrode (DARPA/NASA/NRL VME Program), Modulation @ 10 GHz From Ring Cathodes



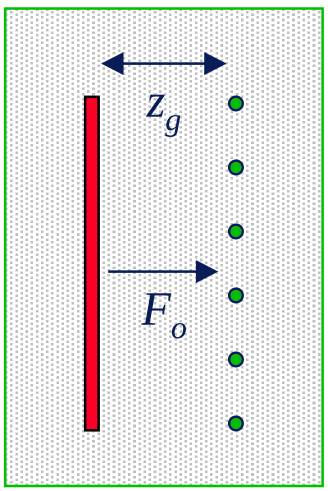
TRANSIT TIME & CUTOFF FREQUENCY

THERMIONIC	Quantity	FIELD EMITTER
2.64 kV/cm	Extraction Field F_o	20 kV/cm
n/a	Tip Field F_{tip}	0.5 V/Å
250 μm	Flight Length z_g	0.77 μm
104 ps	Transit time t	0.096 ps
1.53 GHz	Cut-off Frequency	1667 GHz

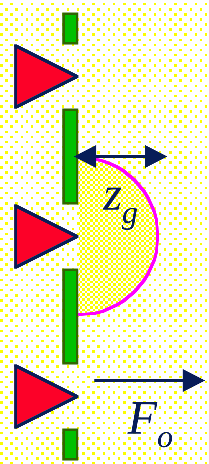
F_{tip} = 0.5 V/Å; V_g = 54.5 V

$z_g = V_g \sqrt{(2 / F_o F_{tip})}$

Thermionic



Field Emitter Array



$$t = \int_0^{z_g} \sqrt{\frac{m}{2V(z)}}\, dz$$

$$V(z) = \frac{F_{tip} z}{F_{tip} z + V_g} V_g \qquad s = \sqrt{\frac{F_{tip}}{2 F_o}}$$

$$t = \frac{\sqrt{\frac{m}{2} V_g}}{F_{tip}} \left(\sqrt{s(s+1)} + \ln\left(\sqrt{(s+1)} + \sqrt{s} \right) \right)$$

CURRENT AND CURRENT DENSITY

Single Tip:

- **SRI**

RF Amplifiers

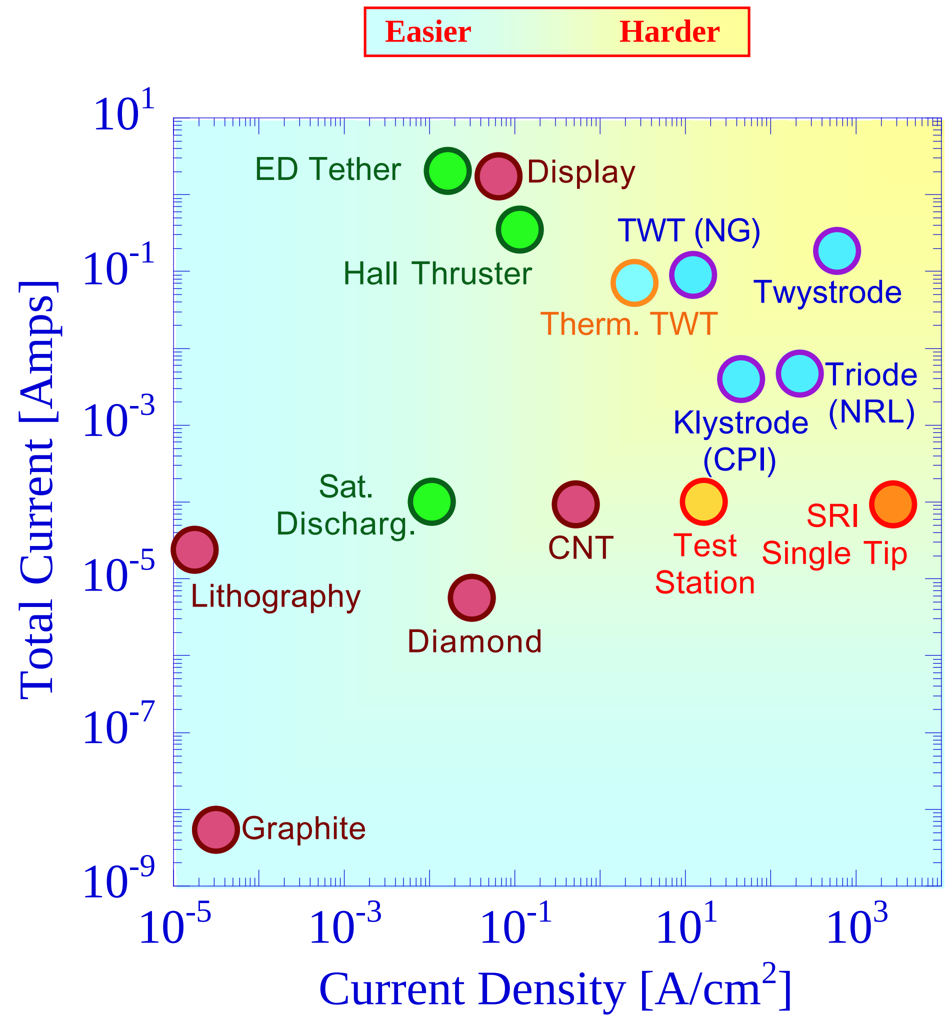
- **Thermionic TWT**
- **FEA-TWT (Northrop)**
- **Twystrode (projected)**
- **Klystrode (CPI)**
- **Microtriode (NRL)**

Space Applications

- **ED Tethers**
- **Hall Thrusters**
- **Satellite Discharging**

Display

- **FEA Display (Motorola)**
- **CNT**
- **Diamond**

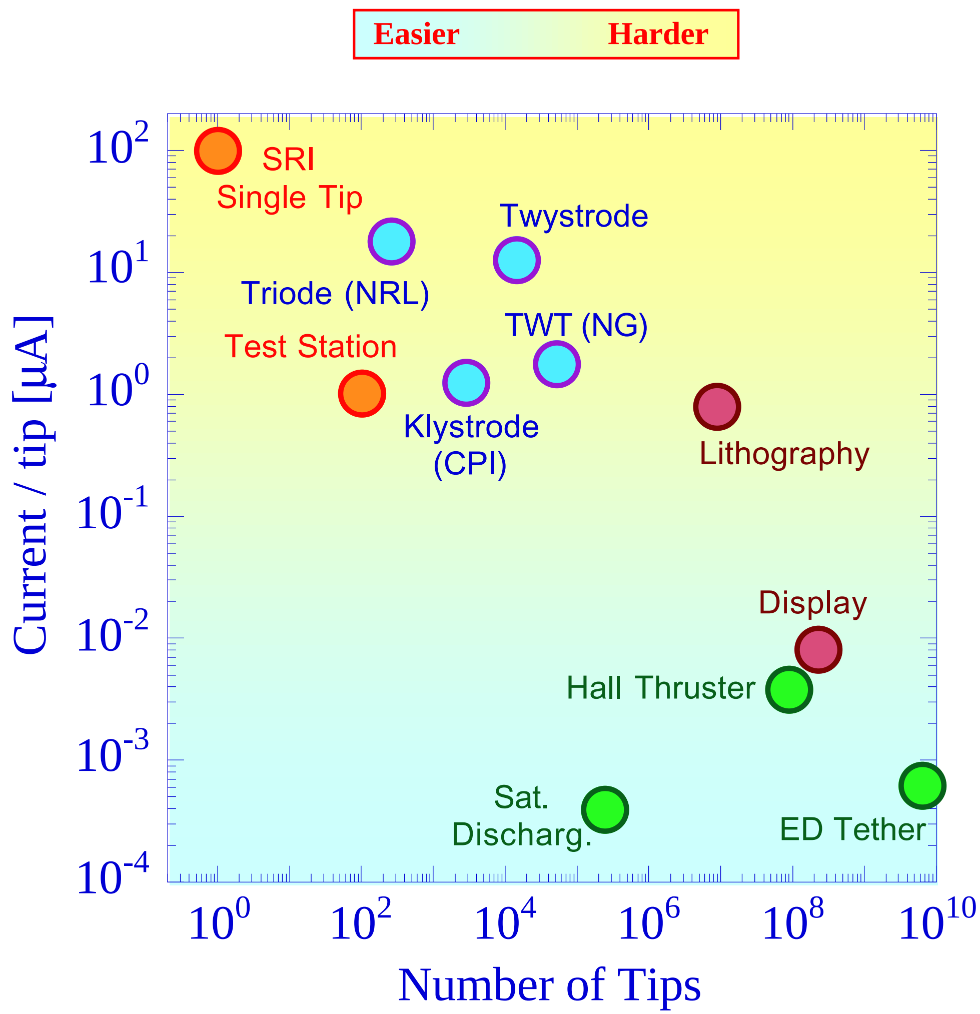


CURRENT PER TIP AND NUMBER

Single Tips Have Been Driven Harder Than Required By Any Application (SRI)

FEA Per-tip Performance In rf Vac. Electronics More Demanding But Require Smaller Area

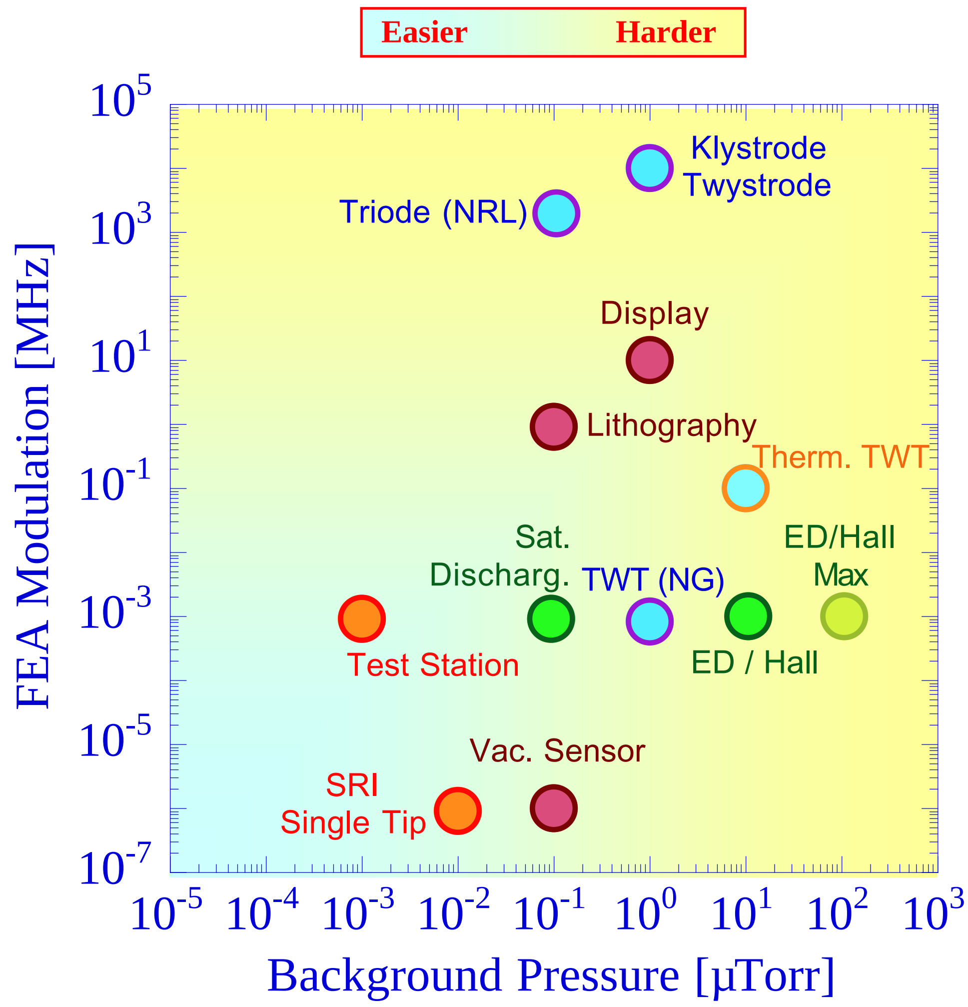
Space Applications and Display Per-tip Performance Requirements Not Large, but Large Areas Required



MODULATION AND PRESSURE

Space-based Field Emitter Applications Must Survive in Environments Far More Challenging Than Other Applications.

Modulation of Electron Beam As for RF Amplifiers Limits Protection Schemes That Can Be Used to Mitigate Arcs



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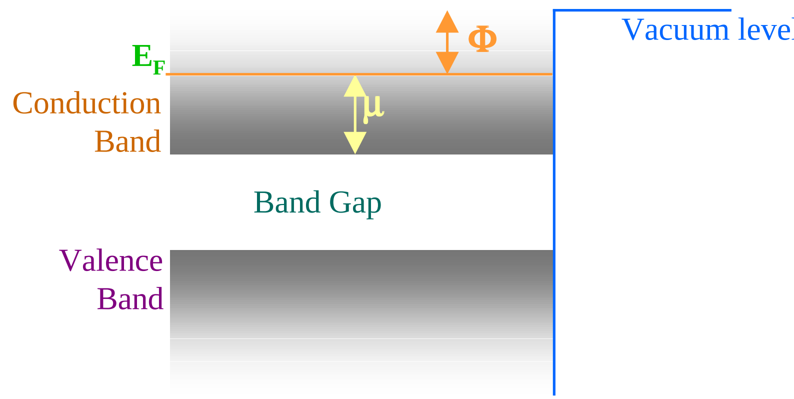
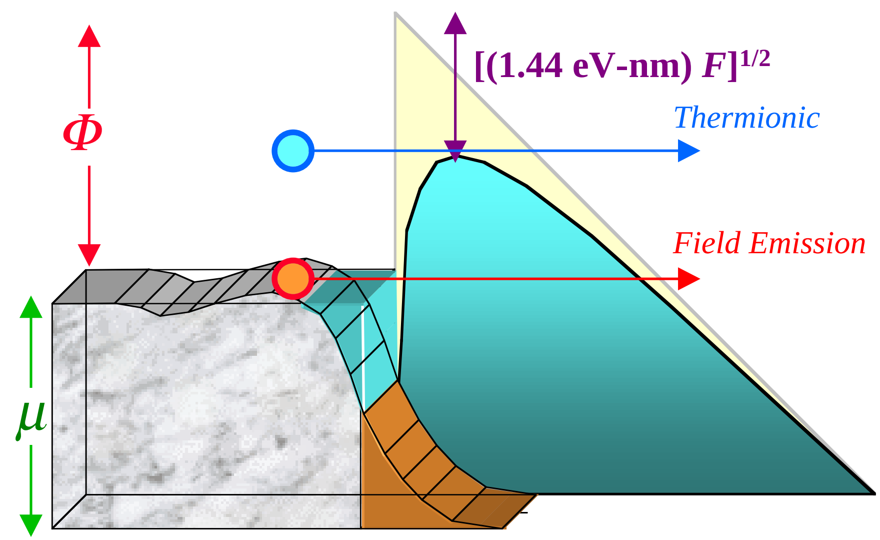
WHAT IMPEDES ELECTRON EMISSION?

FIELD EMISSION FROM METALS

Large Density of Electrons Exist In Conduction Band (> 60 Billion / μm^3); Very Small Fraction Contribute to Current (A/cm^2 ≈ 62 per μs per μm^2)



Most Energetic Electrons Are Several eV Below Vacuum Level: Difference Between Thermionic and Field Emission Is Whether They Pass Over or Tunnel Through the Surface Barrier



Vacuum Barrier Components

$$V_o = -\partial_n [\, n\, \varepsilon_{xc}(n)] - \Delta\phi + \varepsilon_{ion}$$

CHEMICAL POTENTIAL (μ)

Electron Number Density $\rho(\mu)$

$$\rho(\mu) = 2M_c\left(\frac{m}{2\pi\beta\hbar^2}\right)^{3/2}\frac{2}{\sqrt{\pi}}\int_0^\infty \frac{\sqrt{y}}{1+\exp(y-\beta\mu)}dy$$

$$= N_c\frac{2}{\sqrt{\pi}}F_{1/2}(\beta\mu)$$

Zero Temperature ($\mu(0\ ^\circ\mathrm{K}) = \mu_o = E_F$)

$$\rho(\mu_o) = \frac{1}{3\pi^2}\left(\frac{2m\mu_o}{\hbar^2}\right)^{3/2} = \frac{k_F^3}{3\pi^2}$$

Number Density Does Not Change With Temperature, So μ Must:

$$\mu(T) = \mu_o\left[1 - \frac{1}{12}\left(\frac{\pi}{\beta\mu_o}\right)^2 - \frac{1}{80}\left(\frac{\pi}{\beta\mu_o}\right)^4 + \ldots\right]$$

Electrons Incident On Surface Or Interface Barrier Are Distributed In Energy According To A 1-D "Thermalized" Fermi Dirac Distribution Characterized By The Chemical Potential and Called The *"Supply Function"*

$$f(k) = \frac{2}{2\pi^2}\int_0^\infty \frac{2\pi k_\perp dk_\perp}{1+\exp\left(\beta(E_\parallel + E_\perp - \mu)\right)}$$

$$= \frac{m}{\pi\beta\hbar^2}\ln\left[1 + e^{\beta(\mu - E(k))}\right]$$

1-DIMENSIONAL TUNNELING

Schrödinger's Equation: Match ψ and $\partial_x\psi$ at x = 0

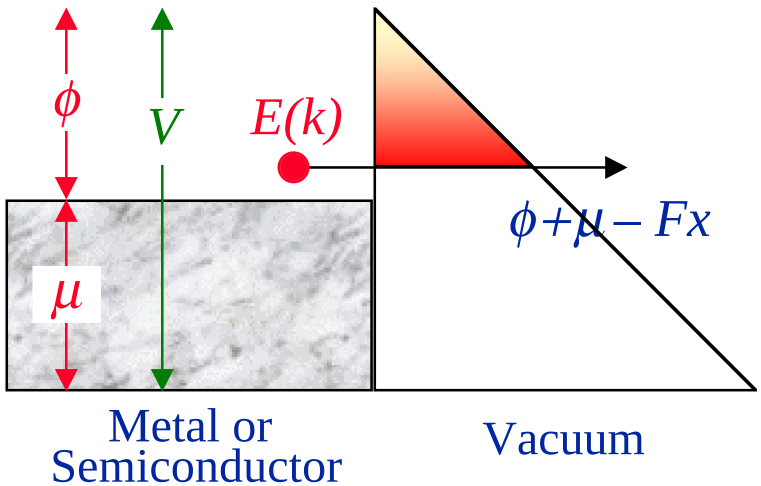
$\psi(k) = c_1\,Ai(z) + c_2\,Bi(z)$ $\quad$ Ai,Bi = Airy Functions; $z = f^{2/3}(v - fx - k^2)$

$$T(k) \approx \frac{16\left(v-k^2\right)^{1/2} k}{4\,v \exp\left(\frac{4}{3f}\left(v-k^2\right)^{3/2}\right) + 8\,\kappa\, k + v \exp\left(-\frac{4}{3f}\left(v-k^2\right)^{3/2}\right)}$$

Where: $f = 2mF/\hbar^2$

$v = 2mV/\hbar^2$

$k^2 = 2mE(k)/\hbar^2$



$\theta(k)$ = WKB Transmission Coefficient as calculated by WKB method

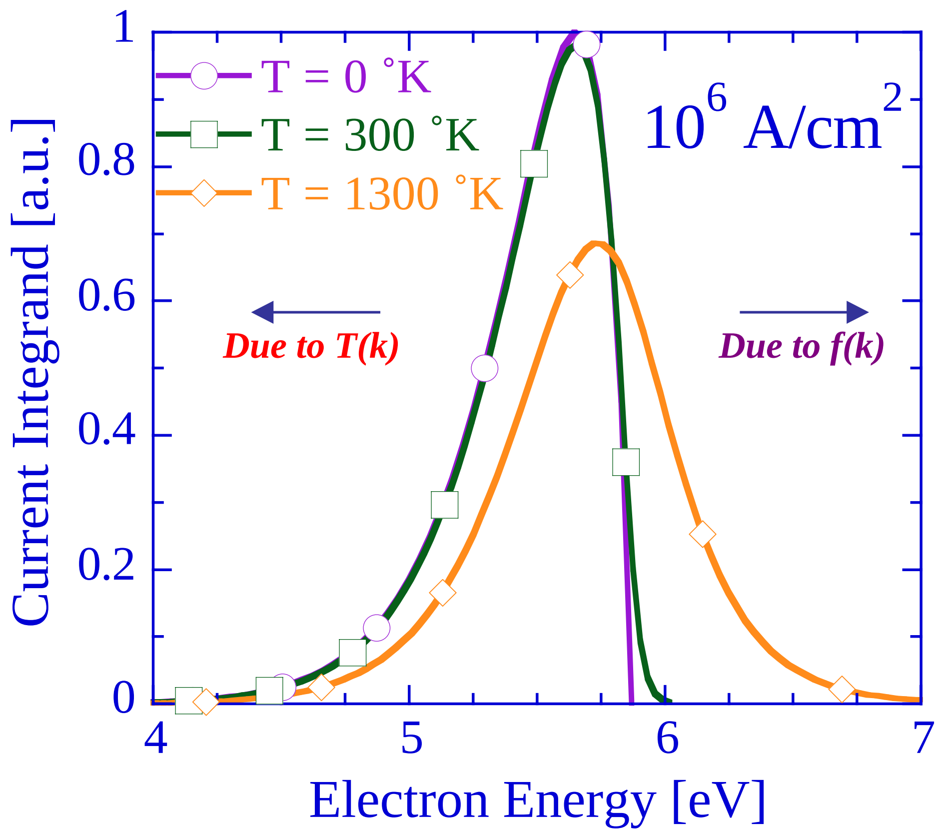
$$T_{wkb}(k) \approx \exp\left(-\frac{4}{3f}\left(v-k^2\right)^{3/2}\right) \equiv \exp\left(-2\theta\left(E(k)\right)\right)$$

ELECTRON DISTRIBUTION

Emitted Current for an Incident Electron With Energy *E* Is Product Of:

- **Transmission Probability**
  - $\Rightarrow$ **Field, Work Function**
- **Incident Supply**
  - $\Rightarrow$ **Temperature, Fermi Energy**

$$J(\beta,\mu,F) = \frac{q}{h}\int_0^\infty T(E)\, f(E)\, dE$$



Energy Distribution of Transmitted Electrons

(molybdenum-like at F ≈ 6 eV/nm)

PROBABILITY OF EMISSION

Product of
Electron Distribution *f(E)* and
Probability of Transmission *T(E)*
Determine Emission Current

The Fowler Nordheim Equation for Field Emission Approximates

$$\ln[T(E)] \approx -b_{fn} - c_{fn}(\mu - E)$$

Value of ln[$T(\mu)$] varies as $\Phi^{3/2}$

Slope of ln[$T(E)$] at μ varies as $\Phi^{1/2}$

Both Vary Inversely With Field (1/F)

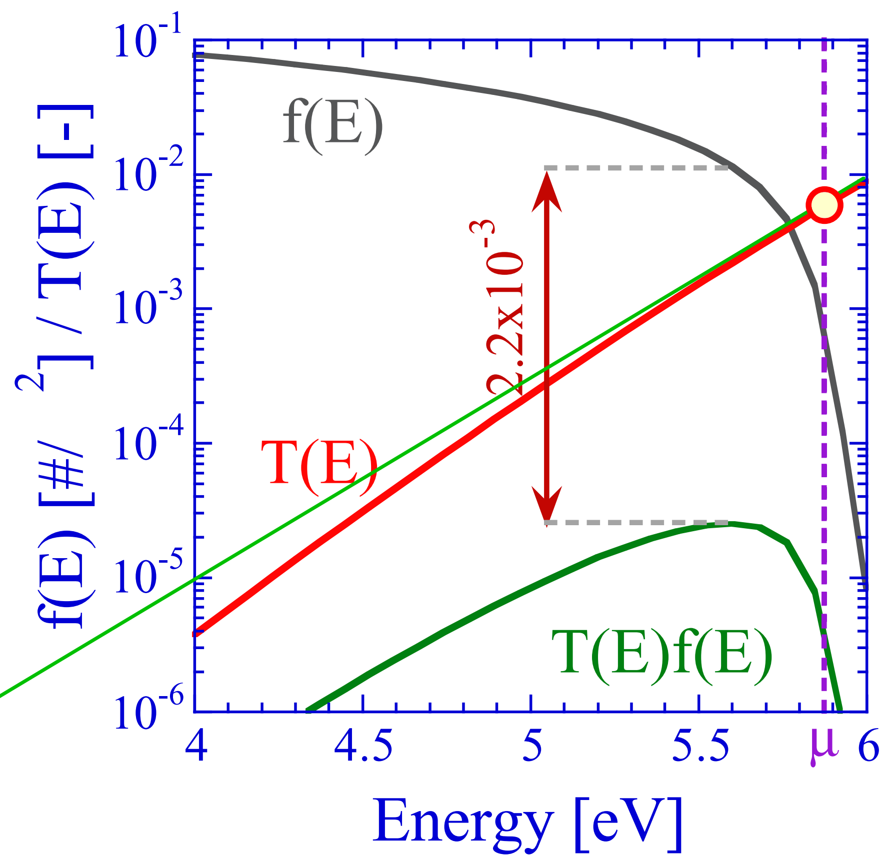


Image Charge Potential
F = 0.6 eV/Å
Φ= 4.41 eV

CURRENT DENSITY

Thermionic Emission (Richardson-Laue-Dushman Eq.)

In Classical Image Charge Model, $V_{max} = \mu + \Phi - \sqrt{(4QF)}$

$$J_{RLD}(T) = \frac{q\,m}{2\beta^2\pi^2\hbar^3}\exp\left(-\beta(V_{\max}-\mu)\right)$$

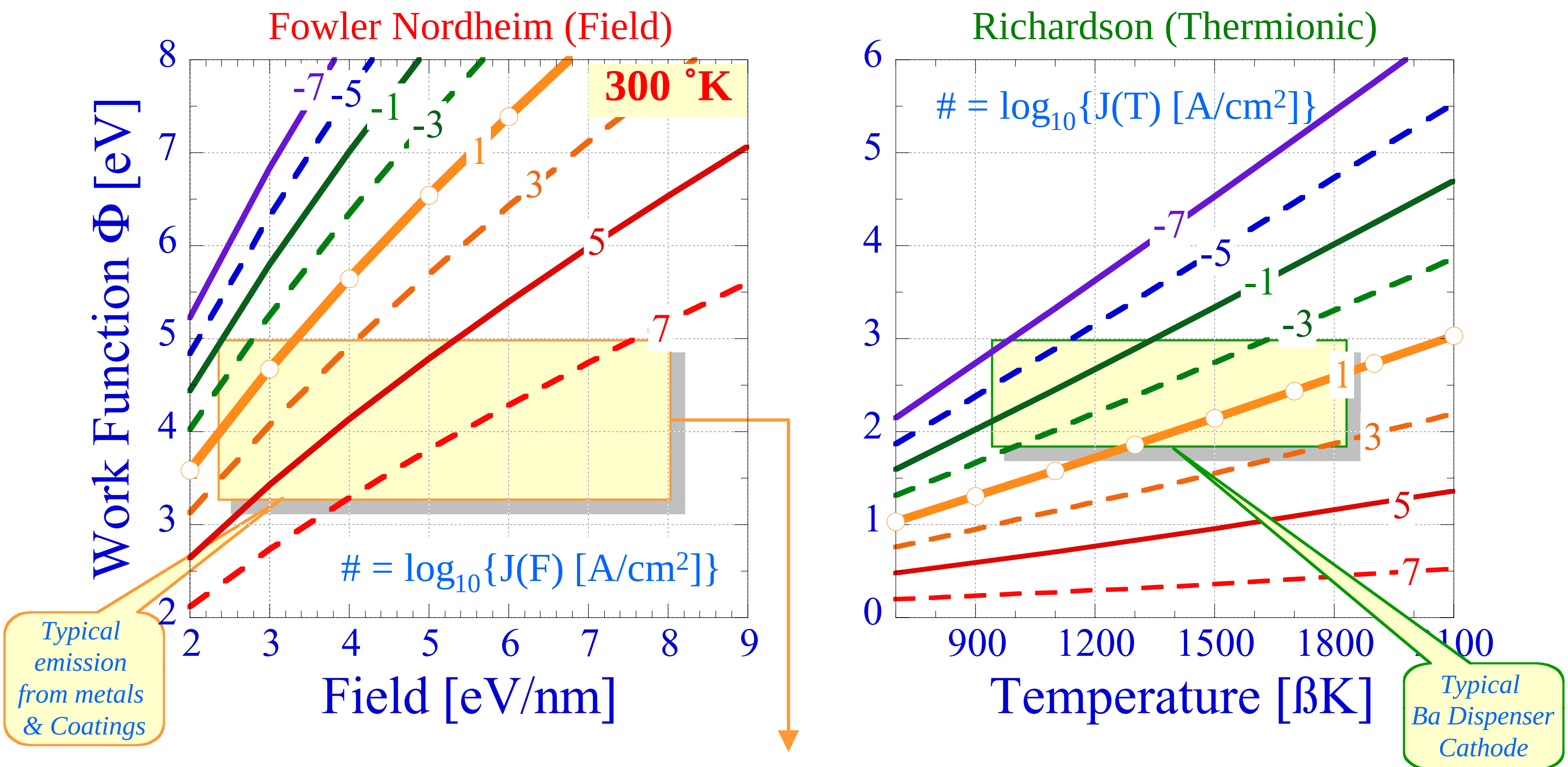
Field Emission (Fowler-Nordheim Equation)

$$J(T,F) = \frac{m}{2\,\pi^2\,\hbar^3\,c_{fn}^2}\exp\left(-\frac{b_{fn}}{F}\right)\left[\frac{c_{fn}\pi/\beta}{\sin(c_{fn}\pi/\beta)} - (1 + c_{fn}\mu)\,e^{-c_{fn}\mu}\right]$$

MODIFICATIONS: To accommodate x_o and x_i, the Work Function in the Fowler Nordheim Equation is replaced with Φ_{eff}, where:

$$\Phi_{effective}(T) = \Phi_o(T) + \frac{8}{3\pi}Q(K_s)\,k_F^3\,x_i^2 + 2\,F\,x_o$$

1-D ELECTRON EMISSION



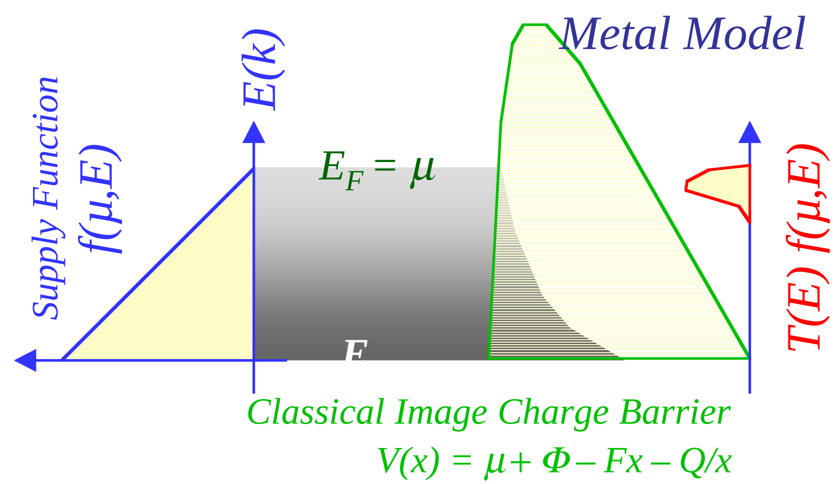
Field Emission: Small Changes in Work Function or Field Result in Orders of Magnitude Change in Current Density:

Low Work Function Coatings + Small Geometry Sought

FOWLER NORDHEIM REVIEW

Metallic Assumptions Which Require Re-analysis for Semiconductors:

- Temperature Is Small and Density Is Constant and High Such That $\beta\mu \gg 1$ and $\mu(T) \approx \mu(0^\circ\ K)$
- Classical Image Charge Potential
- Emission Sharply Peaked About μ Expansion Point in FN-WKB
- Barrier Height Parameter $\phi \approx \Phi$



Metal:	Dominant	Near Unity	Negligible
Semi:	Big	Non-trivial	Not negligible

$$J(F) = a_{fn} F^2 \exp\left(-\frac{b_{fn}}{F}\right)\left[\frac{\pi c_{fn}}{\beta \sin\left(\frac{\pi c_{fn}}{\beta}\right)} - \left(1 + \mu c_{fn}\right)\exp\left(-\mu c_{fn}\right)\right]$$

$$a_{fn} = \left(16\pi^2 \hbar\, \phi\, t(y)^2\right);\quad b_{fn} = \frac{4}{3\hbar}\sqrt{2m\phi^3}\, v(y);\quad c_{fn} = \frac{2}{\hbar F}\sqrt{2m\phi}\, t(y)$$

BAND BENDING

Chemical Potential & Density

$$\rho(\mu) = \underbrace{\left[2 M_c \left(\frac{m}{2\pi\beta\hbar^2} \right)^{3/2} \right]}_{N_c} \underbrace{\frac{2}{\sqrt{\pi}} \int_0^\infty \frac{\sqrt{s}}{1 + e^{s - \beta\mu}} ds}_{F_{1/2}(\beta\mu)}$$

Poisson's Equation

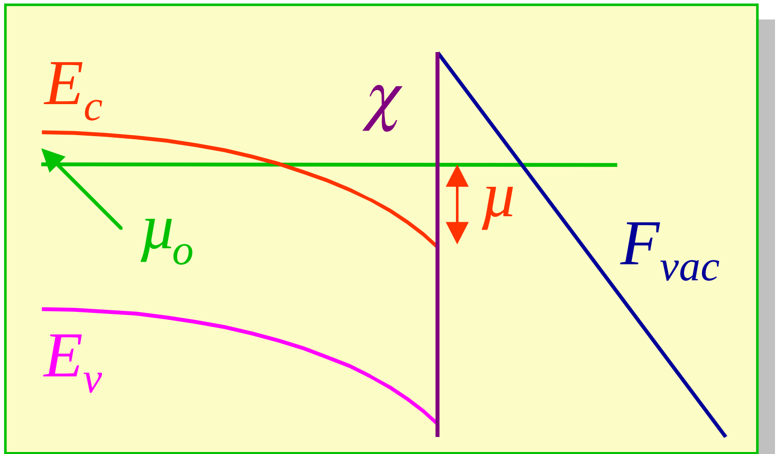
$$\partial_\phi F^2 = \frac{N_c \sqrt{2}}{K_s \pi \varepsilon_o} \left(F_{1/2}\left[\beta(\mu_o + \phi) \right] - F_{1/2}(\beta\mu_o) \right)$$

Asymptotic Case: $\beta\mu \le -2$:

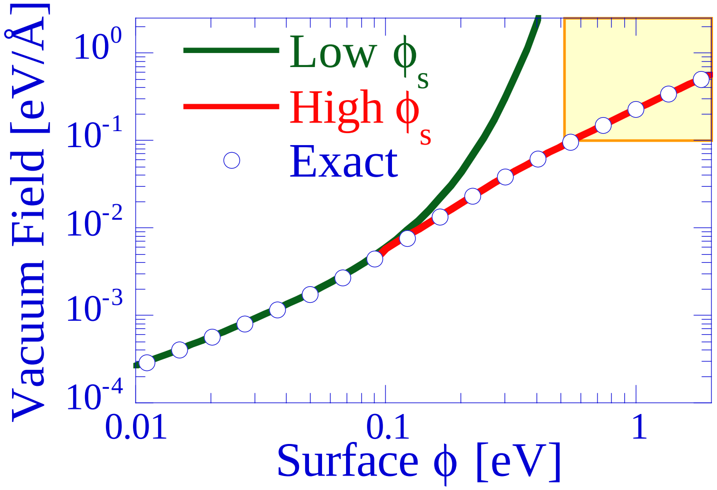
$$\boxed{F(\phi_s)^2 \approx \frac{2 N_c}{K_s^3 \varepsilon_o \sqrt{\pi}\, \beta} \exp(\beta\mu_o) \left[\exp(\beta\phi_s) - \beta\phi_s - 1 \right]}$$

Asymptotic Case: $\beta\mu \gg 1$:

$$\boxed{F(\phi_s) \approx \left(\frac{2\pi^2 N_c}{3\beta K_s^3 \varepsilon_o} \left(\frac{\beta\mu}{\pi} \right)^{1/2} \left[\frac{8}{5} \left(\frac{\beta\mu}{\pi} \right)^2 + 1 \right] \right)^{1/2}}$$



ZECA: $\phi(x)$ is the same as that which would exist if no current was emitted.



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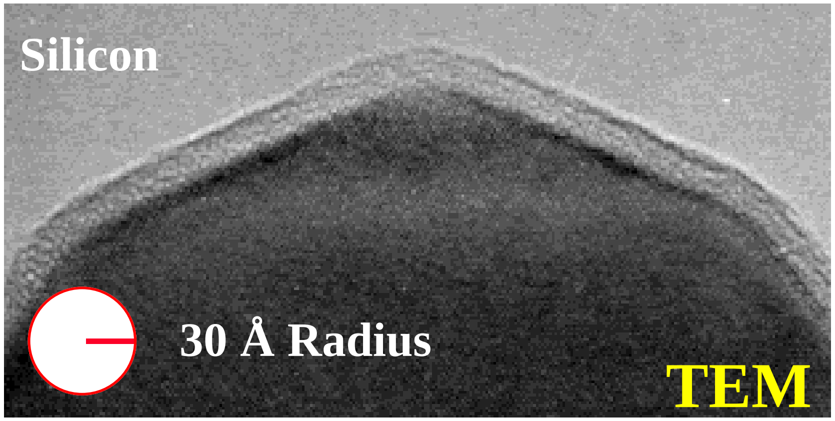
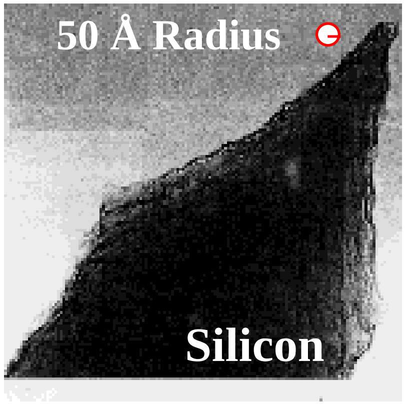
FIELD EMITTER ARRAY TIP SHARPNESS

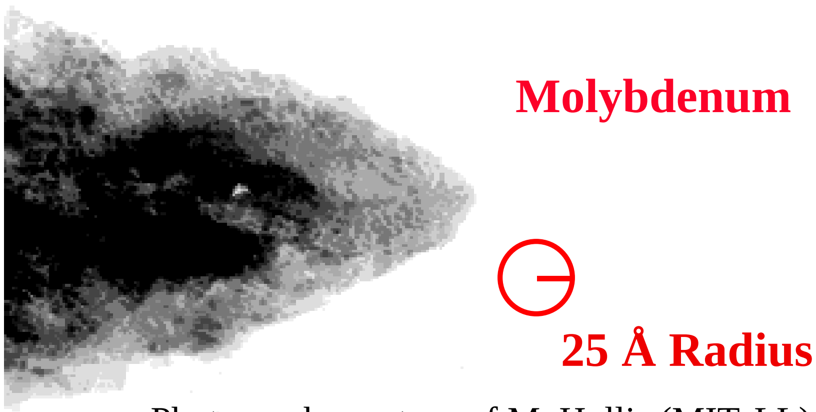
- **TEMs of Various Field Emitter Tips Show Radii of Curvature on the Order of 30–50 Å**
- **Surface Can Have Additional "Structure" Giving Local Field Enhancement Effects**



Photograph courtesy of M. Twigg (NRL)



Photograph courtesy of W. D. Palmer (MCNC)



Photograph courtesy of M. Hollis (MIT-LL)

TIP CURVATURE EFFECTS

Image Charge Barrier *Q* ≈ 3.6 eV-Å

$$V(x) = \Phi + \mu - Fx - Q/x$$

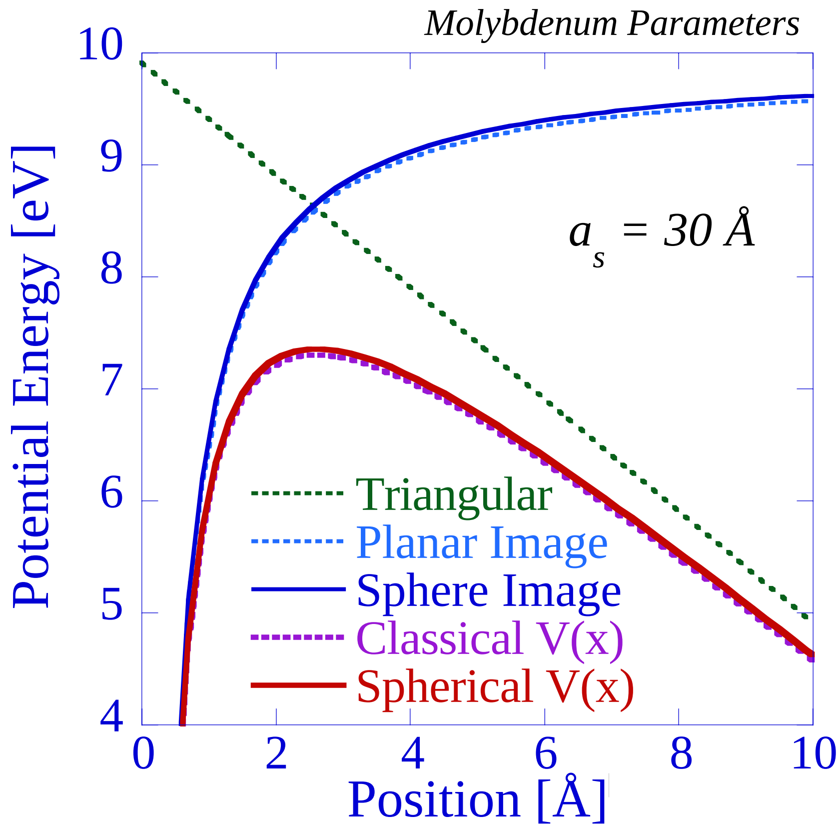
Approximate Tip by Sphere Radius a_s

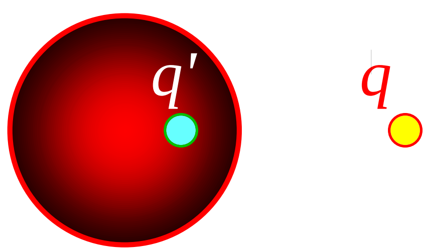
$$\frac{Q}{x} \Rightarrow \frac{2a_s Q}{x(2a_s + x)}$$

To Leading Order, Can Replace:

$$\Phi \Rightarrow \Phi + \frac{Q}{2a_s}$$

$$F \Rightarrow F + \frac{Q}{(2a_s)^2}$$





BOUNDARY ELEMENT SIMULATION

Potential due to Surface Charge Ribbon

$$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\int_\Omega \frac{\sigma(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}d\Omega$$

Discrete Version (M^{-1} = Capacitance Matrix)

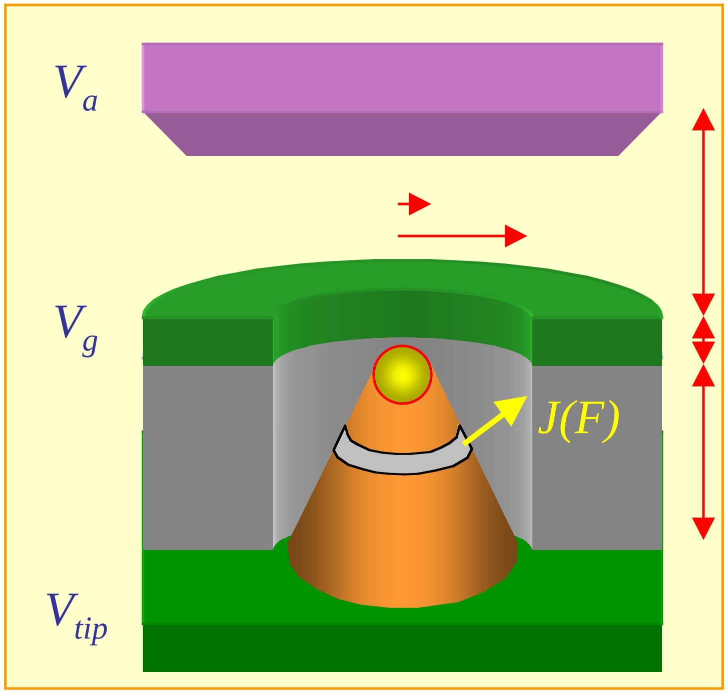
$$\bar{\phi} = \overline{\overline{M}} \bullet \bar{\sigma}$$

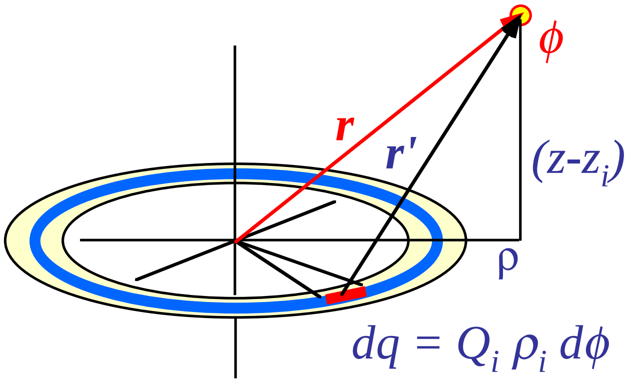
Matrix Sol'n (*K(p)* = Elliptical Integral 1st Kind)

$$M_{i,j} = \frac{1}{\pi\varepsilon_o}\sqrt{1+s_{j+1/2}^2}\int_{\rho_i}^{\rho_{i+1}} \rho' \frac{K(p)}{\gamma_{i,j}}d\rho'$$

Current Vs. Gate Voltage

$$I\left(V_{gate}\right) = \int_\Omega J\left(F(\rho)\right)d\Omega \approx \sum_{n=1}^{N_{tip}} \frac{J(\sigma_{n+1/2})}{\varepsilon_o}d\Omega_n$$





APEX FIELD: SATURN MODEL

Simplest Analytical Model of a Triode Geometry

$$F(a_s,\theta) = 3F_a\cos(\theta) + F_g\,\Sigma\,(2l+1)(a_s/r_g)^l\,P_l(\cos\alpha)P_l(\cos\theta)$$

Where:

a_s = Apex Radius

a_g = Gate Radius

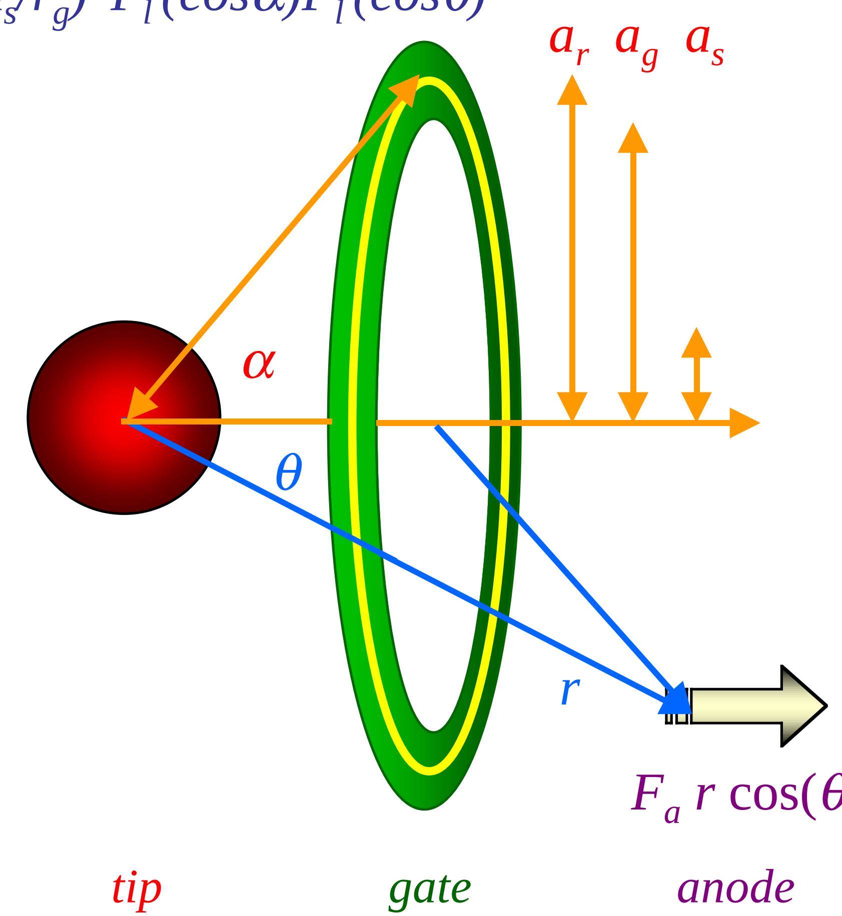
α = Cone Angle

$t = a_r - a_g$

$F_g = Q_g/(r_g a_s)$

FIELD AT APEX

$$F_{tip} \approx V_{gate}\,\pi/a_s \ln(8a_g/t)$$



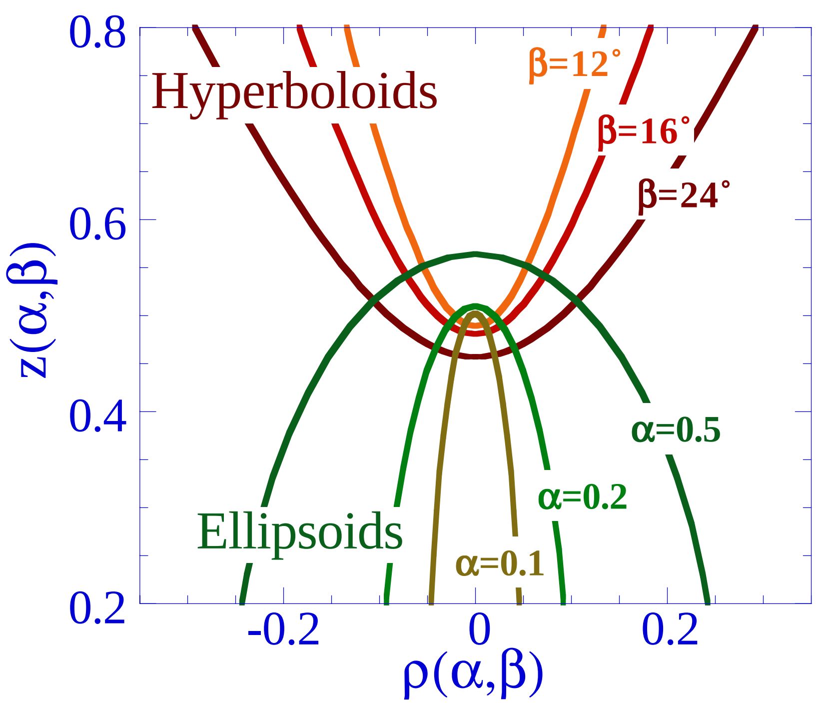
FIELDS: APEX AND SURFACE

Hyperbolic Case (tip specified by β_o)

$$F(\alpha,\beta_o)\Big|_{hyp} = F_{tip}\frac{\sin\beta_o}{\sqrt{\sin^2\beta_o + \sinh^2\alpha}}$$

$$F_{tip} = \left(\frac{\pi}{\ln\left(k\frac{a_g}{a_s}\right)} - \tan^2(\beta_o)\right)\frac{V_g}{a_s}$$

$$k = \frac{1}{54}\left(86 + \frac{a_g}{a_s}\right)\cot(\beta_o)$$



Ellipsoidal Case (tip specified by α_o)

$$F(\alpha_o,\beta)\Big|_{ellip} = F_{tip}\frac{\cosh\alpha_o\sin\beta}{\sqrt{\sin^2\beta + \sinh^2\alpha_o}}$$

$$F_{tip}\Big|_{ellip} = \frac{F_o}{\sinh^2(\alpha_o)\,Q_1\left(\cosh(\alpha_o)\right)}$$

$$\rho = \left(\frac{a_h}{2}\right)\sinh(\alpha)\sin(\beta)$$

$$z = \left(\frac{a_h}{2}\right)\cosh(\alpha)\cos(\beta)$$

FOWLER NORDHEIM PLOT

On an FN Plot, The Relationship Between $\ln\{I(V)/V^2\}$ Is Linear

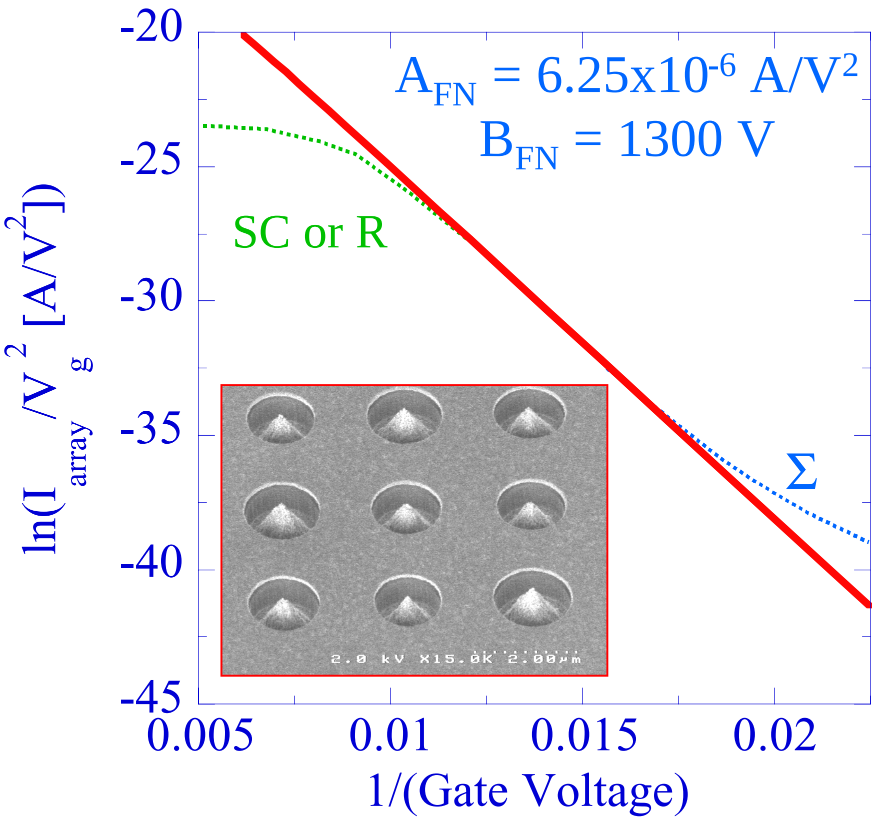
- The Intercept Is Related to A_{FN}
- The Slope Is Related to B_{FN}
- $I(V) \approx A_{FN} V^2 \exp\{-B_{FN}/V\}$

Non-Linearities

- Space Charge (Hi-V roll-off)
- Protective Resistance (Hi-V roll-off)
- Non-uniformity / Statistical Variation in Emission (Low-V convexity)

Simulation Parameters

- Tip radius = 50 Å, Gate Radius = 0.75 μm, Cone Angle = 23˚, Tip-to-Tip = 2.4 μm
  $\Delta s = 0.6$; $\Delta\Phi = 0.165$ eV @ 50% Reduction



Display-like FEA:
60 mA/cm^2 @ 114 V
Photo courtesy of A. Talin, Motorola

STATISTICS

Emission Current Is Dominated By The Sharpest, Lowest Φ Sites Which Therefore Dominate The Distribution.

Assume Linear Distribution In a_s & Φ.

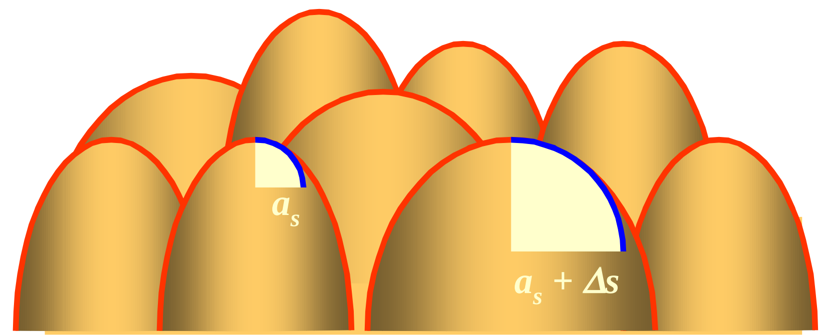
$$I_{tip}(V_g) = b_{area}\left(F_{tip}\right) J\left(F_{tip}\right)$$

$$I_{array}(V_g) = \sum_{j=1}^{N_{tip}} I_{tip}(V_g; a_j, \Phi_j)$$

$$= N_{tip}\, \Sigma(V_g, \Delta s, \Delta\Phi)\, I_{tip}(V_g, a_s, \Phi)$$

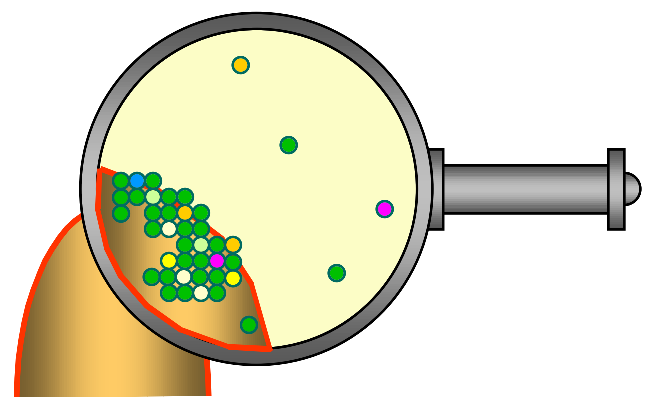
Define Slope Parameter $b_X = \partial I_{tip}/\partial X$

$$\boxed{\Sigma(V_g) = \left(\frac{1 - \exp(-b_a \Delta s)}{b_a \Delta s}\right)\left(\frac{1 - \exp(-b_\Phi \Delta\Phi)}{b_\Phi \Delta\Phi}\right)}$$



Variation in Apex Radius

$\boldsymbol{a(s) = a_s(1 + s)}$



Variation in Work Function

$\boldsymbol{\Phi(s) = \Phi + s\Delta\Phi}$

AREA FACTORS

Definition (Ω = Surface)**:**

$$b_{area}(F_{tip}) = \frac{1}{J(F_{tip})}\int_{\Omega} J(F)\, d\Omega$$

Ellipsoid Approximation:

$$b_{area}(F_{tip})\Big|_{ellipsoid} \approx 2\pi a_s^2 \left(\frac{F_{tip}}{b^o_{fn} + F_{tip}}\right)$$

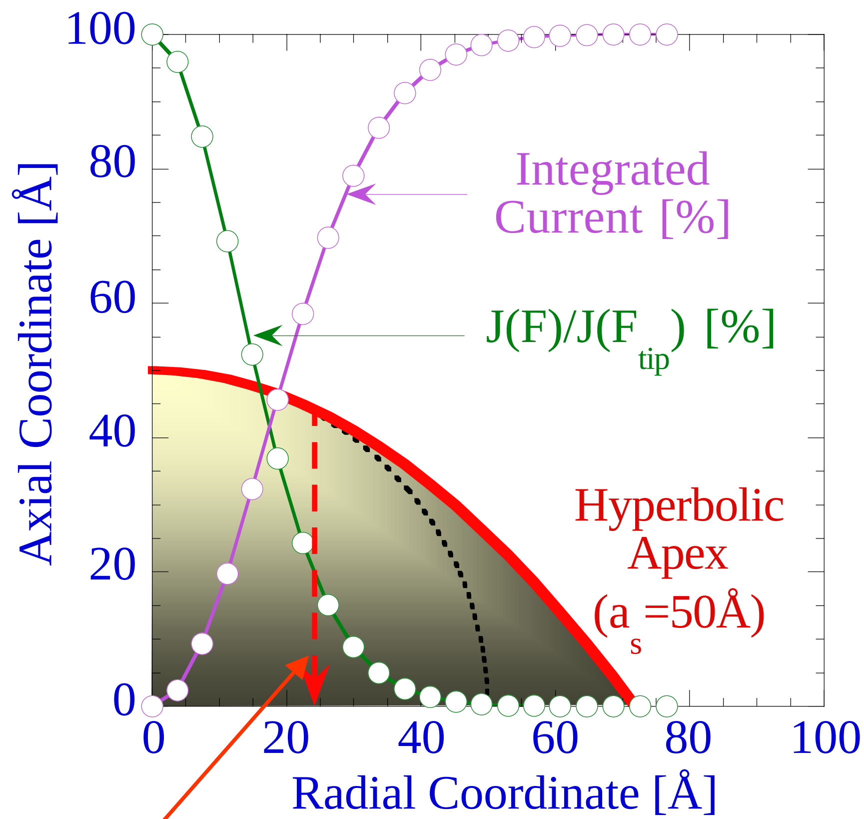
Hyperbola Approximation:

$$b_{area}(F_{tip})\Big|_{hyp} \approx 2\pi a_s^2 \left(\frac{F_{tip}\cos^2(\beta_o)}{b^o_{fn} + F_{tip}\sin^2(\beta_o)}\right)$$

b_{area} < Actual Emitting Area

90% Total Current @ r < 36.5 Å;

99% Total Current @ r < 52.2 Å;



Example:
Moly Hyperbola (15˚)
a_s = 50 Å, F_{tip} = 0.5 eV/Å
$b_{area} \approx \pi\,(2.4\ \mathrm{nm})^2$

USING THE FEA STAT/HYPER MODEL

Extraction of FEA Performance From Experimental Data for Spindt-type FEA

$$I_{array}(V) = N_{tips}\,\Sigma\left(\Delta s, \Delta\Phi; V\right)\, b_{area}\left(F_{tip}\right)\, J_{FN}\left(F_{tip}\right)$$

Red parameters adjusted until *Theory* = *Exp*. A_{FN} and B_{FN}

Statistics	Characteristic Area	Current Density
$a_g, T, \Phi, \Delta\Phi$	Φ	a_g, T, Φ
$\beta_c, a_s, \Delta s$	a_s, β_c	a_s, β_c

FIXED PARAMETERS:

a_g	**Gate Radius**
T	**Temperature**
Φ	**Work Function**
N_{tips}	**Number of Emitters**
$\Delta\Phi$	**Accounts for Degradation in Emission Over Time at Fixed Voltage**
R_{array}	**Gate And/or Array Resistor**
%	**Percent of Emitted Current Intercepted by the Gate**

ADJUSTED PARAMETERS:

a_s	Emission Site Radius Exp & Theory Suggest ≈ 3-7 nm
β_c	Cone Angle, Limited by SEM to 12˚ - 20˚ for Moly
Δs	Tip radius variation parameter, specified by intercept on FN plot Limited by SEM to < 12 for Moly

VARIATION OF SHM PARAMETERS

Variation of the Current *vs* Voltage Behavior With Changes In Geometrical and Material Parameters Is Most Easily Seen Graphically Using the Fowler Nordheim Parameters

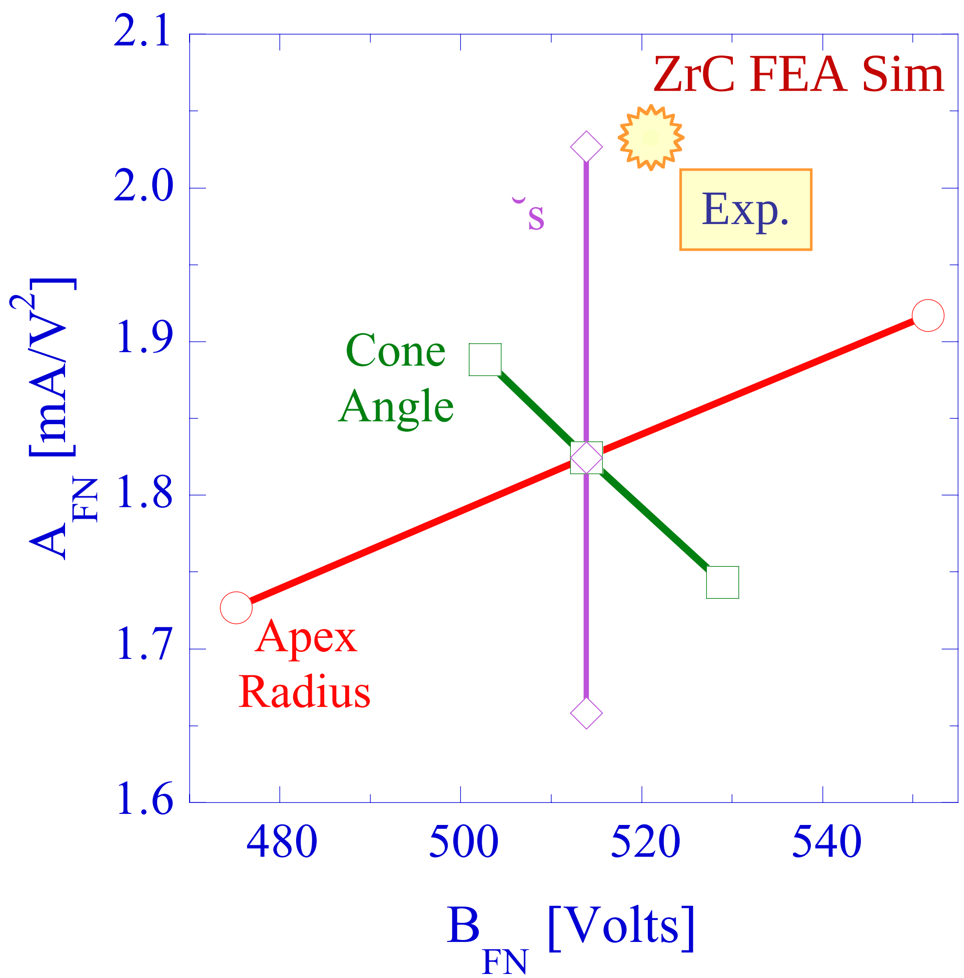
Graphical Analysis Allows the SHM to "Zoom In" on Exp. Values; Work Function and Emission Site Radius Are Greatest Determinants of Performance

To Right: Variation of Dominant Base-line Parameters by ± 10%

(Here: FN Parameters Evaluated Over Fixed Field Regime)

"Baseline" parameters (intersection point):

$\Phi(F\!=\!0) = 3.6$ eV, $a_s = 40$ Å, $a_g = 0.45$ µm, $\beta_c = 15^\circ$, $\Delta s = 2$, $\Delta\Phi = 0.045$ (80% Redux), $N_{tips} = 50$K, $T = 300$ ˚K



Conclusion: Decrease Δs by ≈ 10%

Increase a_s by ≈ 3%

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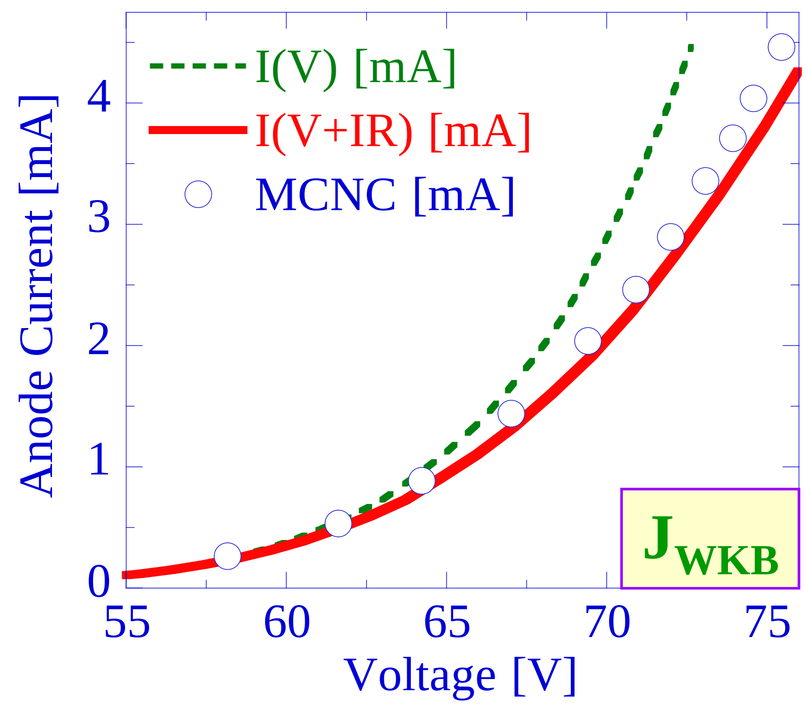
EXTENSION TO SEMICONDUCTORS

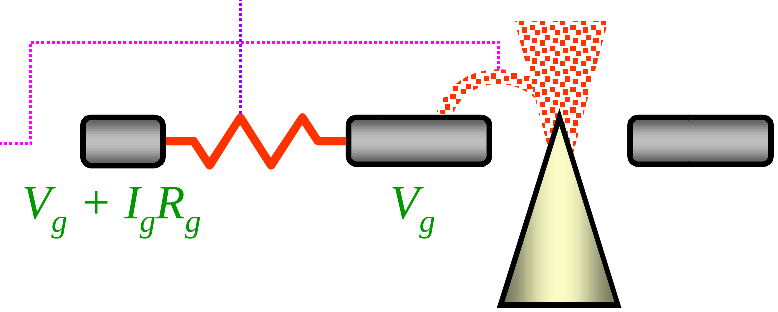
Comparison to Experimental Data:
D. Temple, *et al.*, JVSTA16, 1980 (1998)

Material / Environmental Parameters
- Temperature [˚K] 300.0
- Electron Affinity [eV] 4.05
- Eff. Electron mass [m/m_o] 0.3282
- Dielectric Constant (K_s) 11.9
- # Bands (M_c) 6

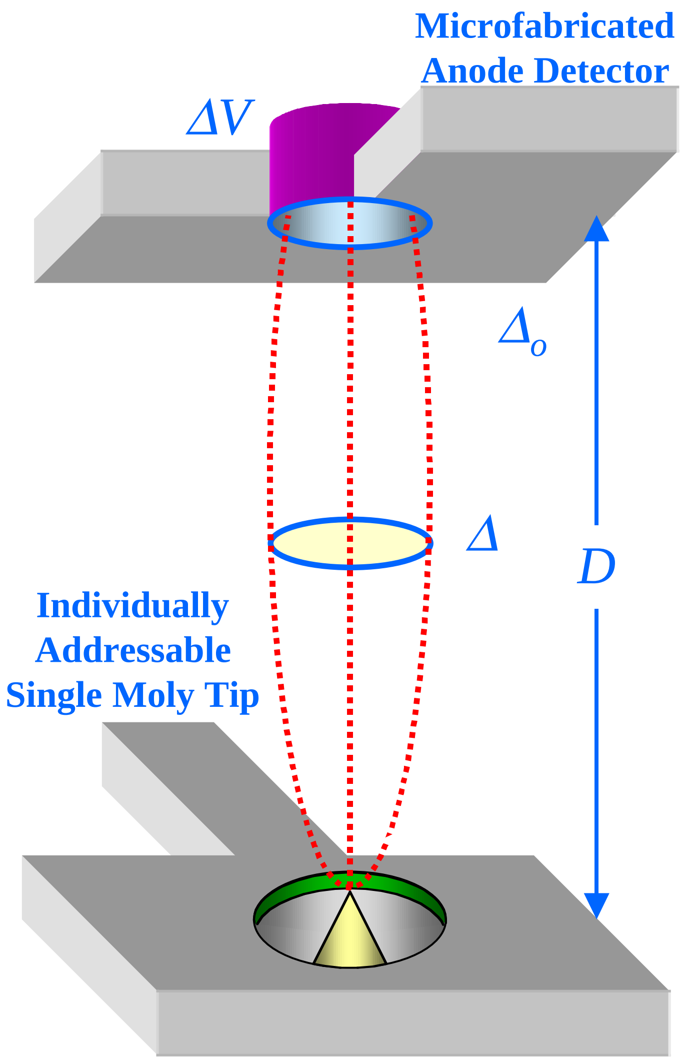
Specified Parameters *(Temple, et al.)*
- Gate Radius [μm] 0.9
- Tip Radius [nm] 2.6
- Cone Half-Angle [degrees] 20.0
- Gate Resistance [kOhms] 100.0
- Number of Tips 18291.0

Assumed Parameters
- Gate Interception Current [%] 0.85
- ***ΔΦ* [eV]** **0.3**
- ***Δs* [-]** **0.2**





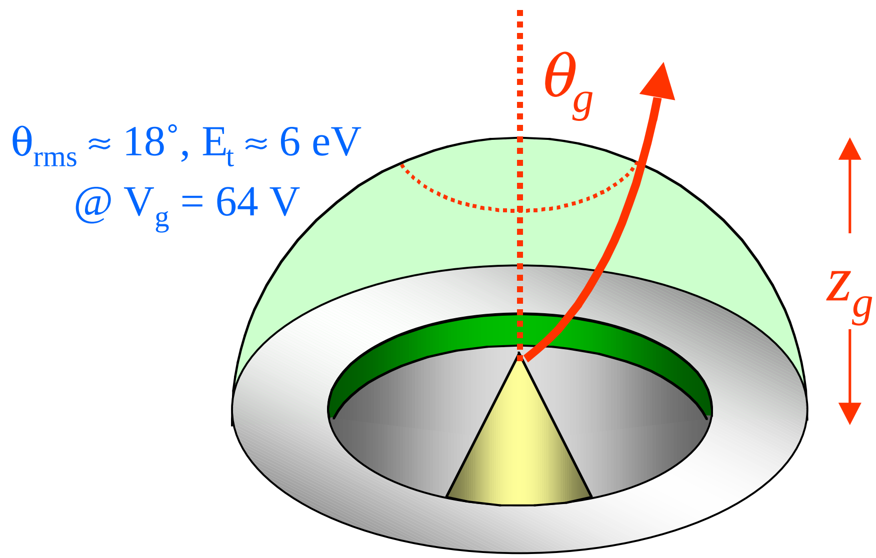
ELECTRON DISTRIBUTION



RMS Emission Angle, Transverse Energy

$$\langle \theta_g^2 \rangle \approx \frac{\sqrt{4\chi+2}}{4\chi}; \quad \frac{E_t}{V_g} \approx \frac{4\chi+1}{16\chi^2}$$

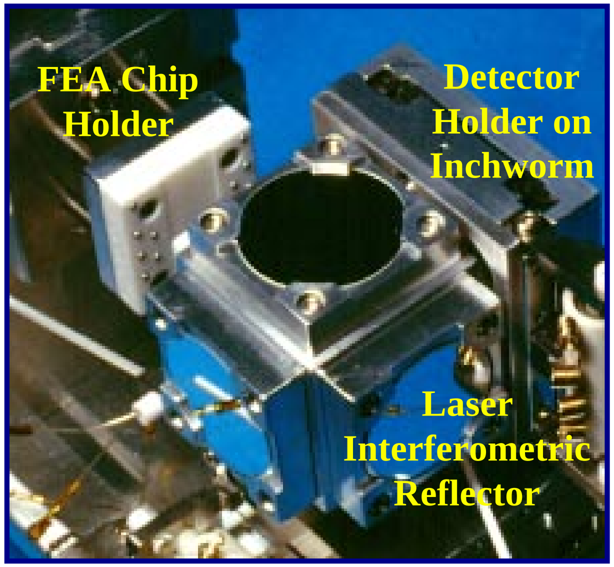
$$\chi = \frac{1}{2}\left(1+\frac{b_{fn}}{2F_{tip}}+\frac{1}{2}\sin^2\beta_c\right)$$

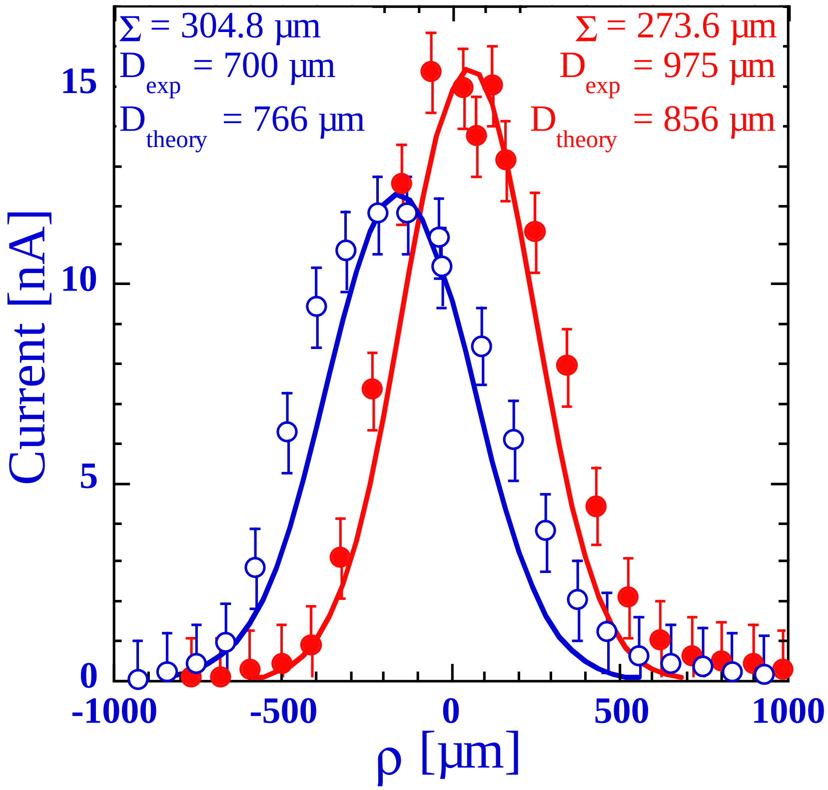


Hemispherical Boss Representation of Emitter

θ_g Refers to Boss Angle

EMISSION DISTRIBUTION EXPERIMENT





- Microfabricated Detector System w/ Laser Interferometric Feed-Back For Nanometric Precision of Movement of Faraday Cup Detector.
- Single Emitter Emission Distribution

Uncertainties:
Current: ± 1 nA; D: ± 100 μm

SPACE CHARGE

Fields:

- Gate to Anode: $F_o = (V_a - V_g)/D$
- Sheet of Charge: $\delta F = 2\pi\sigma\alpha_{fs}\hbar c$

  σ = *charge density* $\approx \kappa\,\Sigma(\Delta s, V_g) / d_{tt}^2$

  $\kappa \leq 1$ *beam spreading factor*

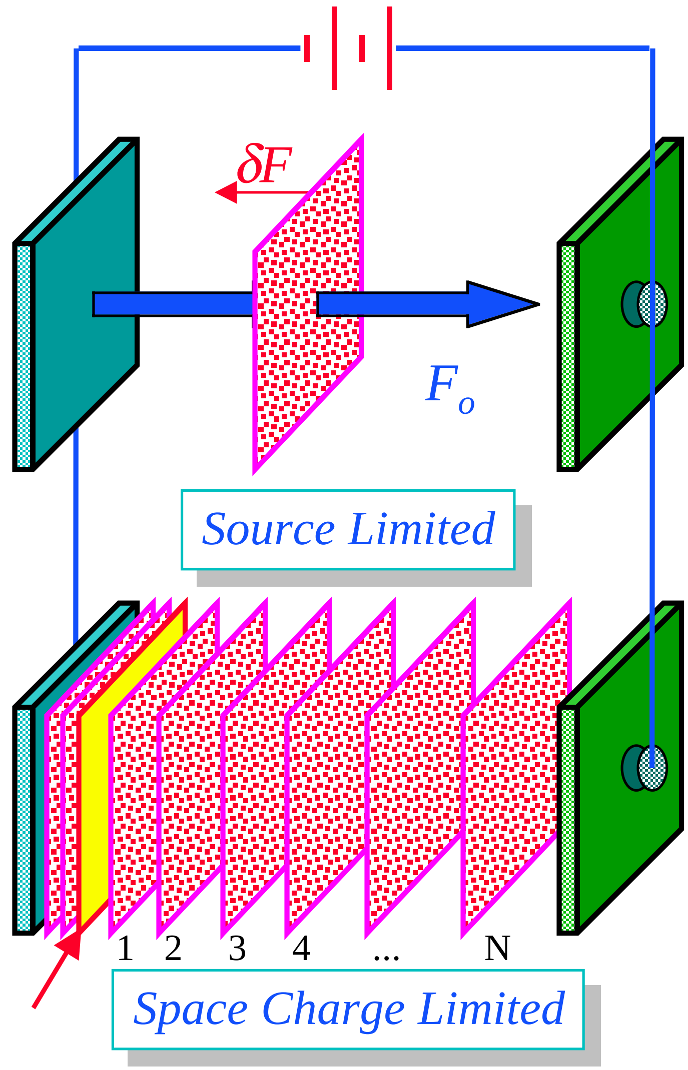
Transit Time (v = velocity):

- Source Limited: $\Delta t_o = 2D/(v_a + v_g)$
- Space Charge Limited: $\Delta t = 3D/(2v_a)$

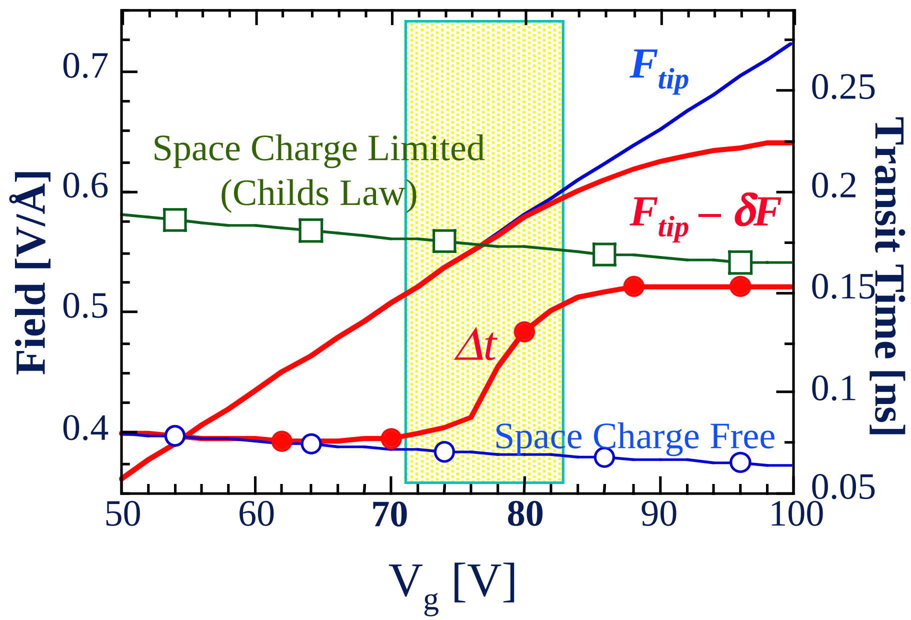
Space Charge Limit (Child's Law)

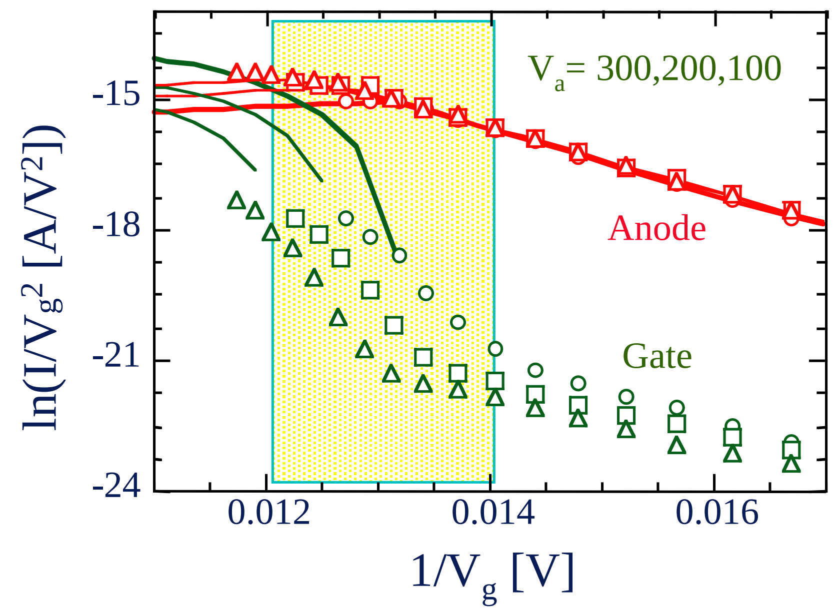
- If $N\,\delta F > F_o$, Sheet Will Eventually Stop, Possibly Return to Gate
- $N = \Delta t\, I_{tip}$ *(in units of eV, fs, Å, e)*

$$I_{Child} = \frac{m\, v_a^3}{18\pi\, \alpha_{fs}\hbar c\, \sigma D^2}$$



TRANSIT TIME AND TIP FIELD





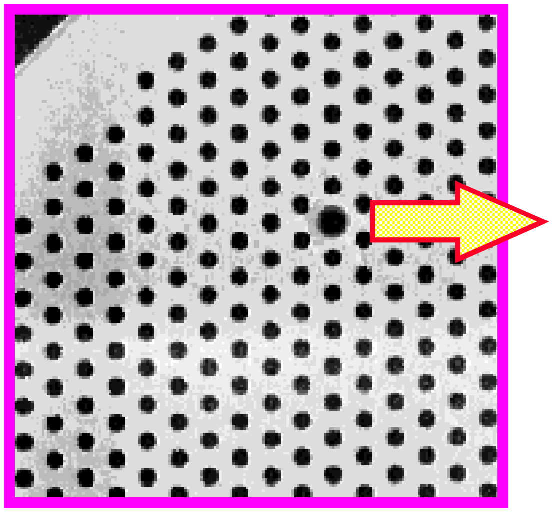
Tip Field Suppression

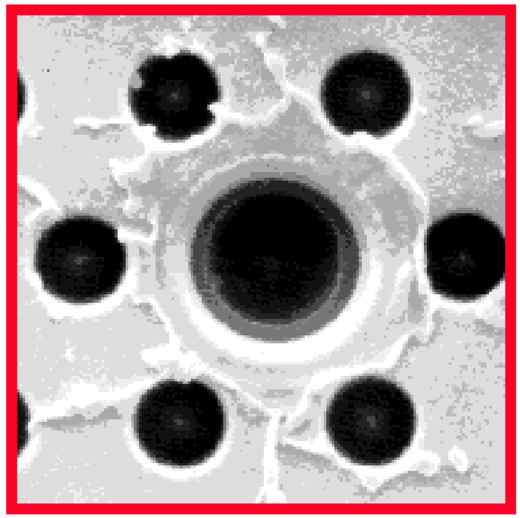
- **Gate to Anode: Number of Sheets Between Gate and Anode: N ≈ $\Delta t / \tau$**
- **Unit Cell: Distance between successively emitted electrons: $L \approx F_{tip}\, \tau^2/2m$**

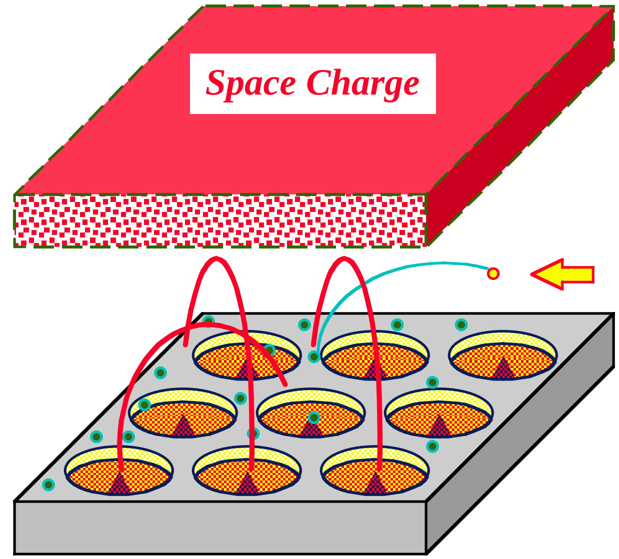
Space Charge

- **Gate to Anode: Current Deflected by Virtual Cathode Gives Rise to Gate Current**
- **Unit Cell: Field Emitted Electrons Collectively Act to Diminish the Tip Field at Emitter Apex**

GATE CURRENT AND TIP FAILURE







- **Insufficient Anode Field Causes Electrons to Return to Gate or to Conducting Surfaces; Nanoprotrusions Direct Electrons Towards Gate**
- **Adsorbates Released, Becoming Ions Which Cause Sputtering Damage and Generate Nanoprotrusions; Excessive Current From a Tip Causes Heating**
- **Tip To Gate Arc Initiated: Metal Melting and Splattered Shorts Tips to Gate**
- **Protection: Resistance to Limit Current During Arc and/or Carbide Coatings With Low Self-Diffusion Rate and Greater Inertness To Contamination**

Photographs Courtesy of W. D. Palmer (MCNC) and J. Shaw (NRL)

PROTECTION SCEMES

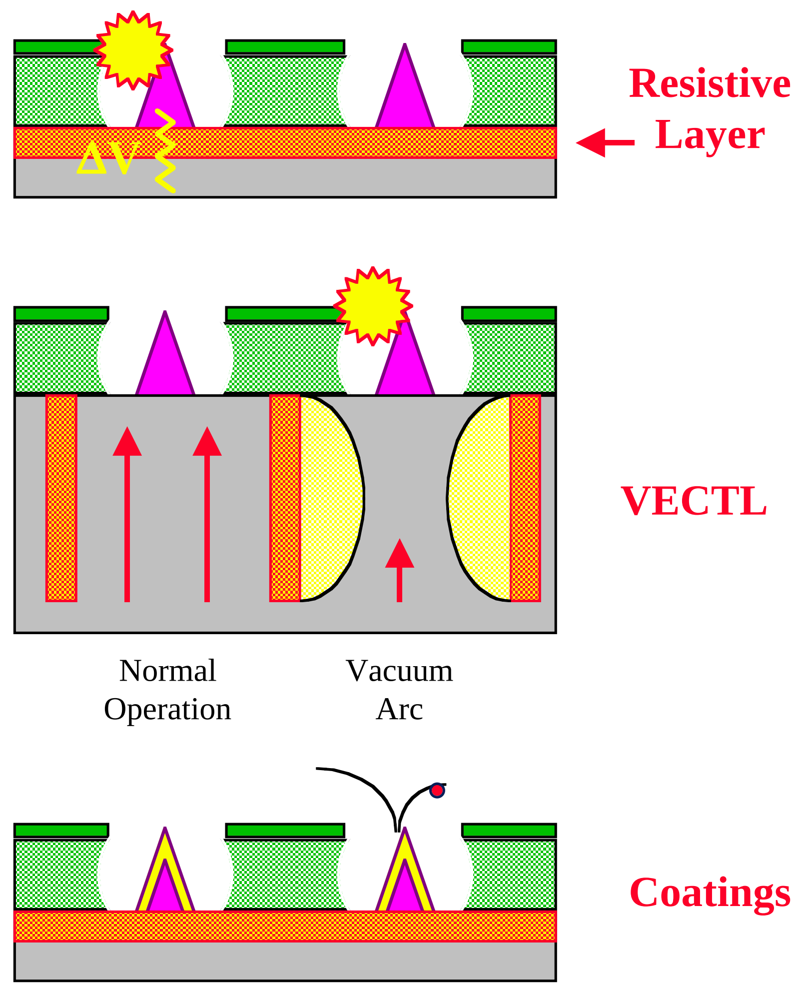
Resistive Layer Schemes Widely Used by FED Community to Protect FEAs With Success; Protection for Rf Applications More Problemmatic But May Be Possible

Simulations Indicate a Reduced-Geometry VECTL-FEA Can Be Modulated at X-Band

L. Parameswaran, C. T. Harris, C. A. Graves, R. A. Murphy, M. A. Hollis, "Resistive Arc Protection for Field-Emitter-Array Cold Cathodes used in X-Band Inductive Output Amplifiers," Tech. Dig. of 11th IVMC, (Asheville, NC, July 19-24, 1998).

Zinc- and Hafnium-Carbide Are Robust, Stable, Low Work Function Coatings

W. A. Mackie, T. Xie, and P. R. Davis, "Transition Metal Carbide Field emitters for FEA Devices and High Current Applications," Tech. Dig. of 11th IVMC, (Asheville, NC, July 19-24, 1998).



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SIMULATION OF I(V) DATA

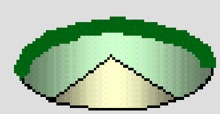
Fowler Nordheim

	A	B	C	D	E	F	G	H	I	J	K
1	Specified Terms	BEGIN: Pick Data Set	Dependent Terms	RESET	I(V) Generator		Restrict Range	FN Generator			
2	Term	Value	Term	Value	Field [eV/Ang]	V+IR	Current [A]	1/V	FN(I)	Linear Fit	Values
3	Data Set:	SRI/CPI	beta [1/eV]	38.6817	0.450	47.46	2.237E-07	0.021071	-23.0327	# of I(V)	21
4	Image Charge a/c	Analytic	Nc [1/Ang^3]	2.5094E-05	0.465	49.04	3.804E-07	0.020391	-22.5674	AFN [A/V^2]	1.5362E-04
5	Material	Metal	kfac [-]	8.7819	0.480	50.62	6.267E-07	0.019754	-22.1317	BFN [V]	675.5
6	Work Function [eV]	3.767	Beta-g [1/Ang]	0.0095	0.495	52.21	1.003E-06	0.019155	-21.7228	Correlation Coef R^2	1.0000
7	Tip radius[Å]	48.7	Chem (0-) [eV]	5.8706	0.510						
8	Gate Radius [Å]	2000	Surf Dens [1/Ang^3]	0.0646	0.525						
9	Cone Angle [°]	15	Chem (0+) [eV]	5.8706	0.540						
10	Dielectric constant	1000000	kf [1/Ang]	1.2413	0.555						
11	Bulk Density [#/Å^3]	0.0646	Xo [Ang]	0.6937	0.570						
12	e- mass/mo	1	Xi [Ang]	-0.2880	0.585						
13	Mc	1	Phi-eff [eV]	5.2227	0.600						
14	Q [eV-Å]	3.5999	J(F) [e/fs-Ang^2]	1.33948E-06	0.615						
15	Temperature [°K]	300	barea [Ang^2]	1274.418489	0.630						
16	Δs	4.100	Sigma [-]	0.020367913	0.645						
17	ΔPhi ?	0.201	Voltage [V]	73.82	0.660						
18	Field [eV/Å]	0.7	V+IR	73.87	0.675						
19	Number of Tips	16000	Array Current [A]	8.9130E-05	0.690						
20	Gate Resistance [MOhms]	0.1	Single Tip ThetaRMS [°]	17.14	0.705						
21	Gate Current [% Anode]	0.5	Single Tip Transverse E [eV]	6.60	0.720						
22		Save Data Set			0.735						
23					0.750						
24											
25											

Greetings / Input Data / NtipCalc / Parameters / Dialog1 / Dialog2 / Dialog3 / Dialog4

Dialog Title

Image Charge type	Classical
Material / Geometry Type	SRI/CPI
Temperature	300
Δs	0.2
ΔΦ	0.3
Gate Resistance [Mohms]	0.1
Gate Current [% of I(anode)]	0.5



The Statistical Hyperbolic / Ellipsoidal Model

? Cancel OK

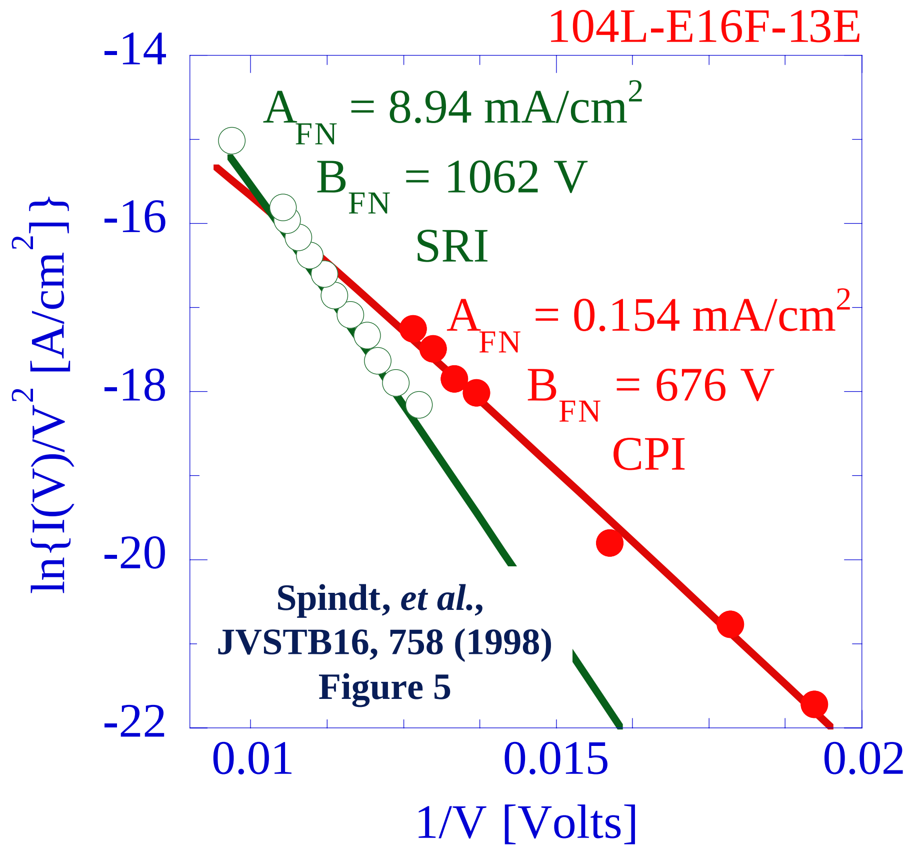
Numerical Models Incorporated Into EXCEL Worksheet for Rapid Characterization of I(V) Data and Parameter Extrapolation

DIFFERENT TEST STATION COMPARISON

SRI Ring Cathodes Measured in Different Test Stations at SRI & CPI

SRI: Sharp + large Δs: small # give most of current;
CPI: Blunt + small Δs: more tips participate

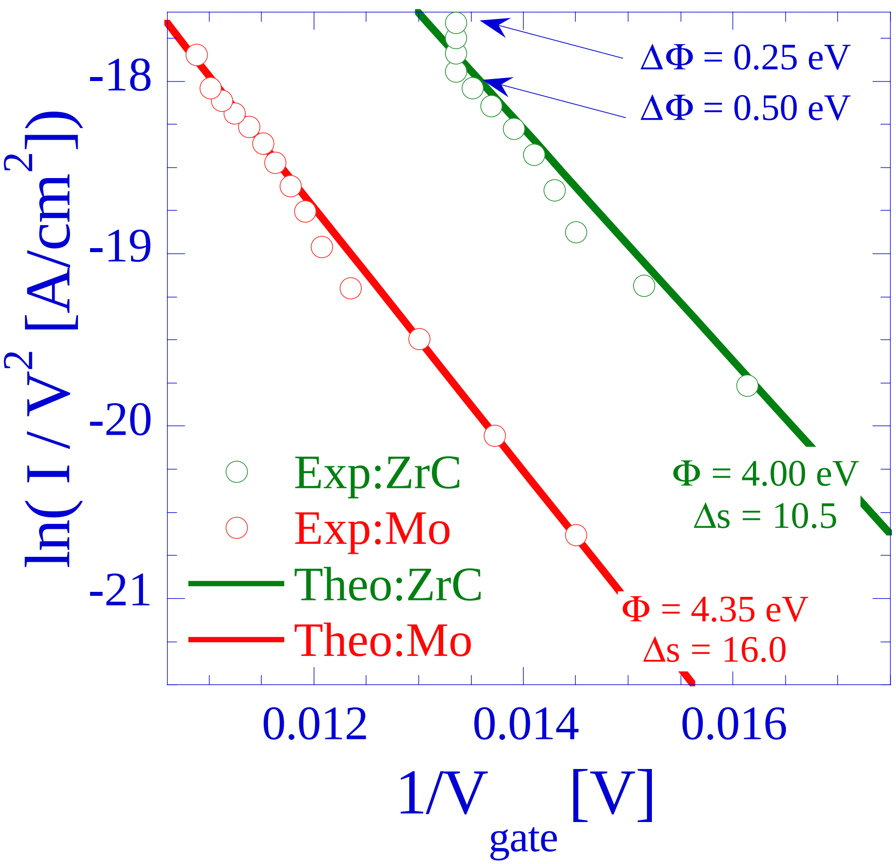
Data Set	***SRI***	***CPI***
Eff. Φ (F=0) [eV]	4.41	4.41
Eff. Tip Radius [Å]	48.7	92.0
Cone Angle [°]	15	15
Gate Radius [µm]	0.2	0.2
Δs [-]	4.1	0.1
$\Delta\Phi$ [eV] @ 50%	0.201	0.175
# Tips	1.6×10^4	3.2×10^4



- **"We Believe That the Data... Reflects Differences in the Average Work Function, ß Factor, and Effective Emitting Area Between the Two Cathodes." "... Galvanic Etching of the Emitter Tips [May Have] Occurred During the Final Cleanup of This Sample." Spindt, *et al.***
- **Theoretical Parameters Very Tightly Constrained - Converged Upon Values Recapitulate Experimental Intuition About Effect of Processing on Arrays**

ANALYSIS OF Mo & Mo/ZrC EMITTERS

Data Set:	SRI1103D	LRI1103D
Image Charge	Analytic	Analytic
Material	Metal	Metal
Tip radius[Å]	**36**	**36**
Gate Radius [Å]	4500	4500
Cone Angle [˚]	18	18
Temperature [˚K]	300	300
Δs	**16.000**	**10.500**
ΔPhi	**0.500**	**0.500**
Field [eV/Å]	0.82	0.6737
Number of Tips	50000	50000
Eff Work Func @ F=0	**4.35**	**3.9974**
% Emit @ F	0.4226	0.561



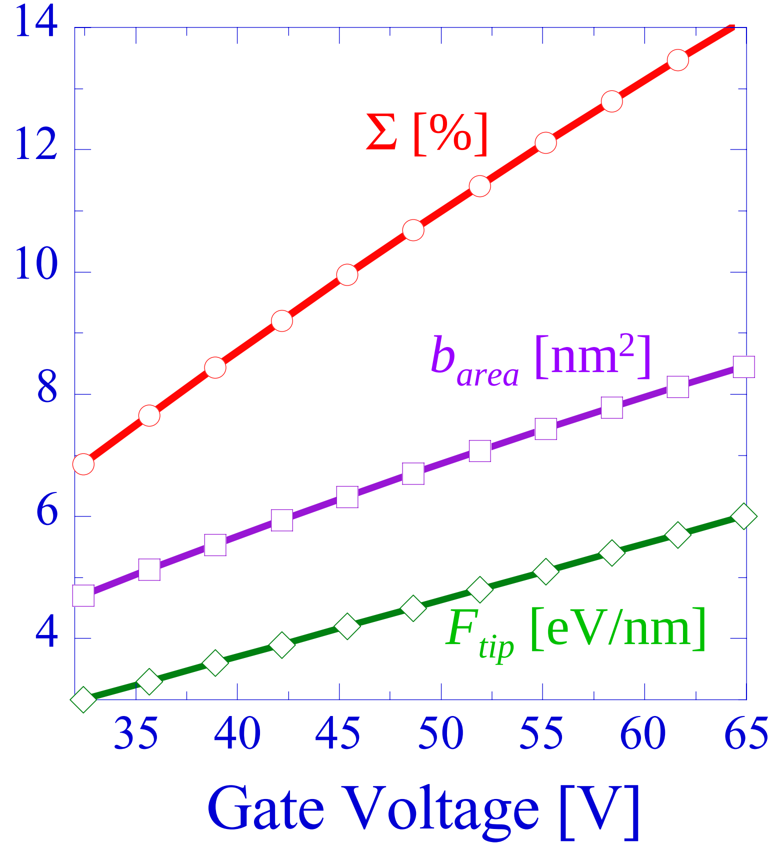
Effect of ZrC Coating under assumption of no change in Δs or a_s:

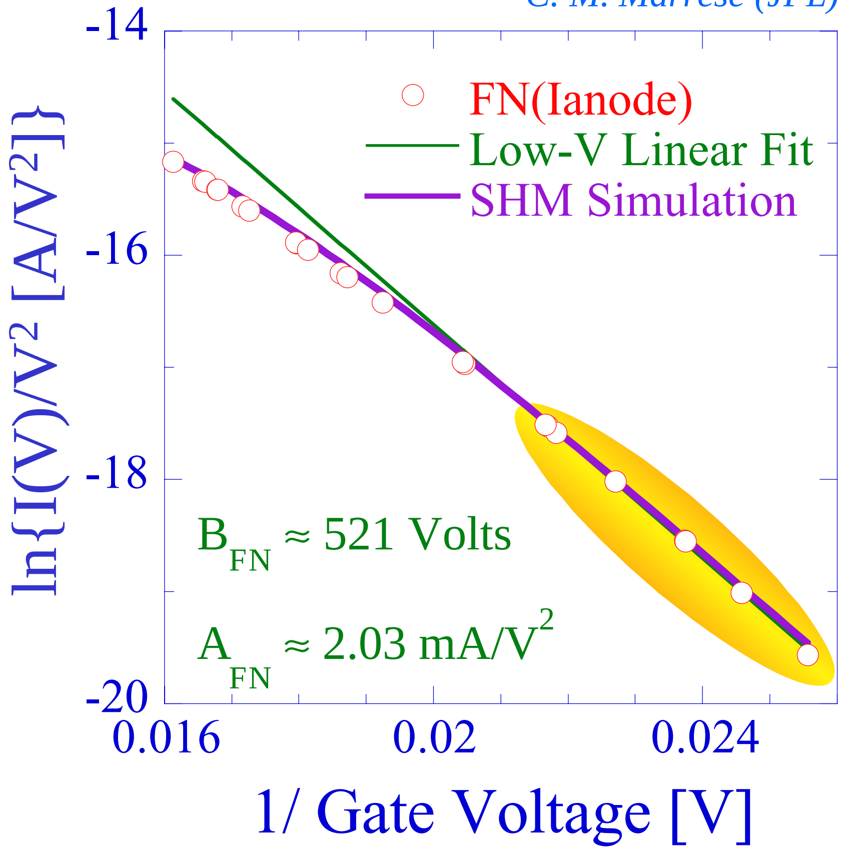
- Improvement of Work function by 0.35 eV
- Improvement of effective radius uniformity: from Δs = 16.0 to Δs = 10.5

ANALYSIS RESULTS

SHM Provides Key Parameters for Array Simulations (LEFT)
As Well As an Account, and Projection, of Experimental Trends (RIGHT)

Experimental Data courtesy of C. M. Marrese (JPL)





CONCLUSION

Tunneling Theory and Application to Model Emission From Field Emitter Arrays and Wide Band Gap Semiconductors

- Directed Towards 3-D Structures and Substrate-Semiconductor Interface Modeling Of General Potentials
- Treatment of Schottky Contacts at Semiconductor Interfaces And/or Non–linear Terms and/or Resonant Effects Possible
- Modeling Effort Directed Towards Characterizing Electron Emission for Subsequent Simulation of Device Performance

Semiconductor Statistical Hyperbolic / Ellipsoidal Model

- Geometry Represented by Hyperbolas and Ellipsoids
- Band Bending, Quantum Mechanical and Many Body Effects Included
- Statistics, Field Enhancement
- Validated by Comparison to Experiment

FEAs & WBG Emitters For

- rf Power Amplifiers
- Space-Based Applications (Electric Propulsion, Electrodynamic Tethers, Satellite Discharging)